

# Running Out of Time: Limited Unemployment Benefits and Reservation Wages

S. Nuray Akin\* and Brennan C. Platt<sup>†‡</sup>

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## Abstract

We study unemployment insurance (UI) in an equilibrium environment in which unemployed workers only receive benefits for a finite length of time. Although all workers have identical productivity and leisure value, the random arrival of job offers creates ex-post differences with respect to their time remaining until benefit expiration. Firms, which are also homogeneous, can exploit these differences, leading to an endogenous wage distribution.

This allows us to examine the equilibrium effect of policy changes in both the size and length of UI benefits. Surprisingly, an increase in benefits can actually cause wages to fall, which is contrary to the predictions of on-the-job-search models. Moreover, we explain well-documented patterns of how the hazard rate of exiting unemployment responds to these policy changes. Our theory also explains why this hazard rate jumps at the time of benefit exhaustion.

Keywords: Unemployment benefits, potential benefit duration, wage dispersion, equilibrium search, hazard rates

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\*Department of Economics, University of Miami, 523-G Jenkins, Coral Gables, FL 33140, (305) 284-1627, [nakin@miami.edu](mailto:nakin@miami.edu)

<sup>†</sup>Department of Economics, Brigham Young University, 135 FOB, Provo, UT 84602, (801) 422-8904, [brennan\\_platt@byu.edu](mailto:brennan_platt@byu.edu)

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# 1 Introduction

A common feature of practically all unemployment insurance (UI) programs is that benefits are only offered for a limited duration. For instance, unemployed workers in the United States typically have been eligible for no more than 26 weeks of payments. Of course, the intent is to discourage moral hazard among workers, who might turn down reasonable job offers if supported by an open-ended benefit.

Indeed, it is not surprising that both the generosity and duration of unemployment benefits would influence employment search decisions. These decisions are often characterized in terms of the worker's *reservation wage*, the lowest wage offer which a worker is willing to accept. A larger benefit would typically increase a worker's reservation wage, as he can afford to be more selective in which job he accepts. As he nears the expiration of his benefits, however, his reservation wage will fall. This result was identified by Mortensen (1977) in an environment where the distribution of offered wages is exogenous. Quite significantly, this creates heterogeneity among otherwise similar individuals; workers who are closer to losing their benefits will be more desperate for a job.

As a consequence, the design of the unemployment insurance system will also affect the firms' incentives in making wage offers. By offering a particular wage, a firm effectively targets workers at a particular point in the unemployment spell. A higher wage will increase the likelihood of acceptance, but will decrease the realized profit if accepted. If these effects precisely cancel one another, identical firms can rationally offer different wages in equilibrium.

Our aim is to characterize the equilibrium effects of limited-duration unemployment benefits on the unemployment rate, wage offer distribution, and the reservation wage distribution. We work with a continuous time environment with infinitely-lived workers who differ only in their realized employment shocks. While unemployed, workers receive a constant benefit for up to  $T$  periods; they also derive constant utility from leisure (or home production) for the entire unemployment spell. Job openings arrive randomly, with wages drawn from an endogenously-determined distribution  $F(w)$ . Unemployed workers formulate a time-dependent reservation wage, taking  $F$  as given.

Meanwhile, firms post wages that maximize expected profits. We assume that the

duration of an individual's unemployment spell is private information;<sup>1</sup> thus, the firm cannot simply offer the reservation wage of the worker it has encountered. Rather, it must weigh the likelihood that a particular wage offer will be accepted, taking the distribution of unemployed workers and their respective reservation wages as given. In one sense, firms are committing to pay a particular wage, being unable to renegotiate if a given job candidate rejects it.

There are two main contributions of the paper. Concerning methodology, we are able to solve this model of wage posting under limited benefits through a particular translation of equilibrium conditions into a differential equation. This innovation allows us to analyze a much richer environment than past models of limited benefits, and thus reproduce many well-documented empirical facts. Concerning policy implications, the equilibrium effects of an increase in benefit size are contrary to those in an on-the-job-search model, and the effects of an increase in benefit duration are contrary to those in simpler models of limited benefits. Thus, the source of wage dispersion is crucial to predict the impact of policy changes. We now consider these contributions in more detail.

The wage dispersion literature has sought to explain why wages vary dramatically among workers, even after controlling for all observable differences in firm or worker productivity. Burdett and Mortensen (1998) propose three potential sources of wage dispersion. Two of these involve unobserved heterogeneity: workers could have unobserved differences in their value of leisure (also identified in Albrecht and Axell, 1984), or firms could have unobserved differences in their productivity. The remaining approach generates dispersion by allowing workers to receive wage offers while currently employed.<sup>2</sup> We study an additional source of dispersed wages: the

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<sup>1</sup>For instance, a worker could be intentionally vague about when he left his former employer. Alternatively, it could be that remaining eligibility for UI is private information even if the unemployment duration is known. Our model actually relies on the former, and UI eligibility often hinges on subtle rules. If so, the firm may not be able to deduce the expiration date from the firing date. Satinger (1998) makes a similar assumption to simplify his analysis in a model of statistical discrimination where workers are homogenous in productivity but differ in quit rate, which is not accurately observed by a firm.

<sup>2</sup>Later work has augmented these models to obtain a closer fit to the observed wage distribution. Postel-Vinay and Robin (2002) allows the current employer to make a counteroffer when workers receive an outside offer. In Burdett and Coles (2003), firms' offers consist of both a wage and a duration. In Christensen, *et al* (2005), workers can influence the job arrival rate through choice of search effort. Carrillo-Tudela (2009) allows the firm to observe the workers' employment status. Chéron and Langot (2010) makes unemployment benefits proportional to the workers prior wage. All of these include on-the-job search; the first also permits heterogeneity in firms and workers.

impending loss of unemployment benefits. In contrast, perpetual benefits (or none at all) would result in a single equilibrium wage and no reason for delay; thus, limited UI benefits effectively create a search friction, providing incentives for those early in their unemployment spell to reject wage offers that would be acceptable at some later date. To isolate the impact of limited benefits, we do not permit on-the-job search or unobserved heterogeneity.

Two previous articles have addressed temporary benefits in simpler environments, chosen due to concerns for tractability of the recursive search problem, which we overcome. In Albrecht and Vroman (2005), unemployment benefits randomly expire at a constant Poisson rate; even the individual worker does not know when his eligibility will expire.<sup>3</sup> This creates two stationary search problems, pre- and post-expiration. That is, all unemployed workers with benefits will solve their search problem identically, whether they have been unemployed one week or 100 weeks, because conditional on still having benefits at that point, the probability of losing the benefits is identical. Similarly, those without benefits solve their search problem independent of their unemployment duration. This binary state variable simplifies the analysis, but also results in no more than two wages being offered or demanded in equilibrium. Our state variable (remaining time until benefit expiration) is continuous, and thus allows a continuum of wages in equilibrium.

In Coles and Masters (2004), unemployment benefits last exactly  $T$  periods. When an unemployed worker encounters a firm, strategic bargaining ensues to determine the wage; and a distribution of wages occurs because a worker's threat point falls as benefit expiration approaches.<sup>4</sup> However, unlike the wage-posting environment in our paper or in Albrecht and Vroman (2005), firms are informed of a particular worker's unemployment duration before an offer is made. This is their crucial mechanism to simplify the recursive problem, but then no offers are rejected in equilibrium and the hazard rate of leaving unemployment is constant throughout the unemployment spell.

In contrast, unemployed workers in our model receive benefits for a fixed period of time, and search for employment in a wage-posting environment. The technical inno-

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<sup>3</sup>In studying the effect of benefit expiration on search effort, Fredriksson and Holmlund (2001) use the same assumption. They assume a single market wage however, rather than investigating potential for dispersion. Cahuc and Lehmann (2000) also investigate the interaction of benefit expiration on search effort assuming a single wage. They use a discrete-time environment where benefits are only received for the first period of an unemployment spell.

<sup>4</sup>Launov and Wälde (2010) investigate workers' choice of search effort in a similar environment. The key difference is that a single wage is determined via collective bargaining.

vation which allows us to solve this model is a translation of the equilibrium conditions into a solvable second-order differential equation governing reservation wages over the unemployment spell. This allows us to use a more appealing set of assumptions and generates richer predictions for individual behavior. For instance, reservation wages steadily decline while exit rates from unemployment steadily increase until benefit expiration, holding constant thereafter. Moreover, our model predicts rising hazard rates of unemployment throughout the benefit period and produces a small jump in the hazard rate at benefit expiration. The empirical literature on the effects of limited UI benefits has largely focused on hazard rates and bears out our model's predictions. In Section 5, we examine these empirical findings in detail.

The second contribution of our work is its analysis of policy decisions in equilibrium, primarily through comparative statics. Indeed, the source of wage dispersion is of vital concern for policy choices. For instance, when wage dispersion arises from on-the-job search as in Burdett and Mortensen (1998), an increase in unemployment benefits will raise wages and compress their distribution. In our model, however, a benefit increase will actually decrease wages and expand their distribution. Larger benefits encourage workers to delay their acceptance of jobs; as a consequence, a higher concentration of unemployed workers are closer to their benefit expiration date. Firms can exploit this fact by offering lower wages as increasingly desperate workers lower their reservation wage.

Additionally, our model allows us to examine the effect of changes in potential benefit duration. We show that extending benefits will expand the wage distribution upward (while leaving the lowest wage unchanged) and increase the unemployment rate. While the policy change allows workers to delay accepting jobs, the deadline also moves farther away. The net effect is that fewer workers reach the expiration of benefits, and wages rise in equilibrium. This contrasts with the prediction of Albrecht and Vroman (2005), where a longer expected duration of benefits lowers the low wage and leaves the high wage unchanged. To allow quantitative comparison of our model's predictions to prior empirical work, we calibrate our parameters to match key statistics of the US labor market, using CPS data.

We consider this model to be most applicable to low skilled workers. First, wage posting is more appropriate for this demographic than wage bargaining (which our model does not consider). Second, this group is more likely to be reliant on unemployment benefits. Higher skilled workers may have accumulated assets to help

smooth consumption, but this is less likely to occur among low skilled workers with associated low wages. Finally, this group is more likely to experience unemployment spells between jobs, rather than making a direct job-to-job transition (which would require on-the-job search).

We proceed as follows: in Section 2, we present the environment and define equilibrium. In Section 3, we solve for equilibrium and characterize its basic features. Section 4 provides analysis of policy changes via comparative statics (including changes in benefit size, benefit length, and job destruction rates), which are then compared against empirical findings in Section 5. Section 6 offers some extensions to the model, incorporating risk aversion, taxation to finance benefits, and endogenous firm entry. Finally, we conclude in Section 7. All proofs are presented in the appendix.

## 2 Model

Consider a continuous time environment with a unit-measure of infinitely-lived workers who are ex-ante identical but only differ in their realized employment shocks. If employed at a given wage, they remain employed at the same wage until the job is dissolved, which occurs randomly at Poisson rate  $\delta$ . For simplicity, we assume linear utility. Let  $\rho$  denote the discount rate.

When workers become unemployed, they receive an unemployment benefit  $b$  for up to  $T$  units of time. They also receive an exogenous utility from leisure (or home production),  $x$ , that continues for the full duration of unemployment. Job offers arrive randomly at Poisson rate  $\lambda$ . If an offer arrives, the particular wage is drawn randomly from the distribution of wage offers,  $F(w)$ . The worker then accepts or rejects that offer, with no recall being allowed.

These decisions will be characterized in terms of a reservation wage,  $R(s)$ , where  $s$  is the remaining time until a worker loses his unemployment benefit. Because of random luck in when an acceptable offer is received, an endogenous distribution of unemployed workers will emerge, represented by cumulative distribution  $H(s)$ . Thus,  $H(0)$  denotes the fraction of unemployed workers whose benefit has expired, while  $H(s)$  denotes the fraction who have  $s$  or less time until expiration. Let  $u$  denote the fraction of the population currently unemployed.

The distribution of offered wages is also endogenously determined from the choices of profit maximizing firms. Any worker who is successfully hired will produce  $p$  dollars

of value each instant. When it encounters a worker, a firm cannot observe  $s$ ; thus it makes an offer that balances the probability of acceptance (taking  $R(s)$  and  $H(s)$  as given) against the realized profit  $p - w$  if it is accepted. We exclude on-the-job search, since this is known to produce wage dispersion.

We assume throughout that  $p > x$ . If this were violated, it would be inefficient to have anyone work, since their marginal product is less than their value of leisure. Additionally, we require the following two conditions:

**Assumption 1.**  $e^{-\frac{T\rho}{2}} > \frac{\rho}{\lambda}$ .

The first assumption ensures that job offers arrive with sufficient frequency to justify some degree of delay. On a technical level, it ensures that our endogenous wage distribution is well-behaved; if it failed, a degenerate distribution would likely emerge.

**Assumption 2.**  $p - x \leq b \left( 1 + \left( \frac{\delta + \rho}{\lambda} - e^{-\frac{T\rho}{2}} \right) e^{-\frac{2\lambda}{\rho} \left( 1 - e^{-\frac{T\rho}{2}} \right)} \right)$ .

This second assumption compares the net social benefit of employment to expected benefits (with a particular discounting to reflect the probability of either ending the spell before exhausting benefits, or losing benefits thereafter). Section 3 explains the genesis of this necessary assumption, but its intuition is simple: if benefits are too low, workers would be inclined to accept offers very early in their unemployment spell, making it more difficult to sustain a continuous wage offer distribution.<sup>5</sup> Moreover, our calibration of the model to US data (presented in Section 5) easily satisfies both assumptions; other developed nations tend to provide even more generous benefits.

## 2.1 Bellman Equations

The worker's decisions are presented recursively, with  $V_e(w)$  representing the discounted present utility of a worker who is employed at wage  $w$ .  $V_u(s)$  is the same for an unemployed worker who has  $s$  time remaining until benefit expiration.

A worker employed at wage  $w$  consumes his wage each period until job separation randomly occurs. At that point, he is entitled to full unemployment benefits<sup>6</sup> and his utility changes by  $V_u(T) - V_e(w)$ .

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<sup>5</sup>The model can be adapted to accommodate low benefits; a sketch of the solution appears in Appendix A.6.

<sup>6</sup>Most UI systems include a minimum employment duration before a worker is eligible for full

$$\rho V_e(w) = w + \delta (V_u(T) - V_e(w)) \quad (1)$$

Next, consider an unemployed worker whose benefits have expired. He receives leisure utility  $x$  each period until he receives an acceptable job offer; thus, his recursive problem is:

$$\rho V_u(0) = x + \lambda \left( \max_{R(0)} \int_{R(0)}^{\infty} (V_e(w) - V_u(0)) dF(w) \right). \quad (2)$$

Finally, an unemployed worker with  $s$  time until expiration has a similar recursive problem with two differences. First, he also receives benefit  $b$  each period. Second, the remaining time until benefit expiration becomes a state variable in this recursive problem. This state deterministically falls each instant, causing his utility to change at rate  $-V'_u(s)$ .

$$\rho V_u(s) = b + x - V'_u(s) + \lambda \left( \max_{R(s)} \int_{R(s)}^{\infty} (V_e(w) - V_u(s)) dF(w) \right) \quad (3)$$

By defining the Bellman equation in this way, we are assuming that both  $V_u(s)$  and  $V'_u(s)$  are in  $C^1$ , *e.g.* continuously differentiable. Thus, we do not examine possible equilibria with discontinuous value functions, focusing instead on “smooth” equilibria. Even though benefits stop abruptly at  $s = 0$ , the present value of remaining benefits declines smoothly as that time approaches.

Equation 1 reveals that  $V_e(w)$  is increasing in  $w$ . Thus, unemployed workers will solve their utility maximization with a reservation wage such that  $R(s)$  will satisfy

$$V_e(R(s)) = V_u(s). \quad (4)$$

By substituting Equation 1 into Equation 3, one obtains  $V_u(s)$  as a differential equation, with Equation 2 as the boundary condition.

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benefits. Similarly, workers who voluntarily quit typically are ineligible for benefits. Our model does not incorporate either feature, but they can be worked in. Ineligible workers would transition directly to  $V_u(0)$  on leaving employment.  $V_e(w)$  would also depend on time remaining until requalification, but would not be complicated to solve since no choices occur during that waiting period.



## 2.2 Steady State Conditions

Consider the flows of workers between states of employment and unemployment. The expression  $(1-u)G(w)$  denotes the measure of workers currently employed at or below wage  $w$ , where  $G(w)$  is the cumulative distribution among employed workers at each wage. Similarly,  $uH(s)$  denotes the measure of unemployed workers with  $s$  or less time until benefit expiration, where  $H(s)$  is a cumulative distribution of unemployed workers. We require that the flows between these states balance so that  $G$ ,  $H$ , and  $u$  remain constant over time. Let  $h(s)$  and  $g(w)$  denote the respective probability density functions.

To allow for the possibility of atoms (*i.e.* discontinuous upward jumps) in the cumulative distribution function, we use the following notation for a mass of workers employed at a particular wage:

$$\mu_G(w) = G(w) - \lim_{\epsilon \rightarrow 0} G(w - \epsilon). \quad (5)$$

The notation similarly applies for a mass of firms offering a given wage,  $\mu_F(w)$ .

The transition from employment to unemployment is simply stated as

$$uh(T) = \delta(1 - u). \quad (6)$$

That is, the  $1 - u$  employed agents become unemployed at rate  $\delta$ , and enter unemployment with full benefits.

Next consider those who still have  $s$  time until expiration. A flow of  $\lambda(1 - F(R(s)))h(s)$  workers will receive acceptable wage offers each instant and enter employment; thus the change in the density of unemployed workers is:

$$h'(s) = \lambda(1 - F(R(s)))h(s). \quad (7)$$

A positive measure of workers might not receive an acceptable job offer within  $T$  periods; thus, we allow  $H(0) \geq 0$ . The reservation wage of those whose benefits have expired,  $R(0)$ , is the lowest of any worker; thus it is also the lower bound of the support of  $F$ . In other words, workers without benefits accept any wage offered in equilibrium, so  $\lambda H(0)$  of them find jobs over a unit of time. To maintain steady state, these must be replaced by the flow of workers whose benefits have just expired,  $h(0)$ . Thus,

$$h(0) = \lambda H(0). \quad (8)$$

Among the employed at any given wage  $w$ , jobs are lost at rate  $\delta(1-u)g(w)$ . These must be replaced by someone who receives and accepts a job at that wage, which occurs at rate  $\lambda u f(w)H(R^{-1}(w))$ , where  $R^{-1}(w)$  denotes the inverse of the reservation wage function, and  $f(w) = F'(w)$ . Thus, the steady state equation becomes:

$$\delta(1-u)g(w) = \lambda u f(w)H(R^{-1}(w)). \quad (9)$$

If  $F$  has an atom at  $w$ ,  $G$  would need an atom at the same  $w$ . Equation 9 would still be required, replacing  $g(w)$  and  $f(w)$  with their respective atoms,  $\mu_G(w)$  and  $\mu_F(w)$ .

### 2.3 Profit Maximization

The steady-state profit of a firm offering wage  $w$  is defined as the realized profits,  $p-w$ , times the average number of workers employed at  $w$  per firm offering  $w$ ,  $\frac{g(w)}{f(w)}$ . The definition similarly applies when an atom occurs at  $w$ , only using  $\frac{\mu_G(w)}{\mu_F(w)}$ . Substituting for this fraction using Equation 9, we can state profits as:

$$\pi = \frac{\lambda u}{\delta(1-u)}(p-w)H(R^{-1}(w)). \quad (10)$$

In equilibrium, any wage in the support of  $F$  should produce the same steady state profit. Note that  $H(R^{-1}(w))$  is the probability that a randomly-drawn unemployed worker will accept wage  $w$ . Thus,  $(p-w)H(R^{-1}(w))$  is the expected per-period profit of a firm that offers  $w$ , from the perspective of a firm that has just encountered an unemployed worker.

### 2.4 Equilibrium

A steady state search equilibrium consists of firm profit  $\pi$ , a reservation wage function  $R(s)$ , a measure of unemployed agents  $u$ , and distributions of employed workers  $G(w)$ , unemployed workers  $H(s)$ , and firms wage offers  $F(w)$ , such that:

1.  $R(s)$  maximizes utility for an unemployed worker with  $s$  time until benefit expiration, given  $F(w)$ .

2. All wages in the support of  $F$  produce the same profit  $\pi$ , while all other wages produce no more than  $\pi$ .
3.  $G$ ,  $H$ , and  $u$  satisfy steady state conditions in Equations 6 through 9.

We note that the combination of Equations 1 and 4 imply that  $V_u(s) = \frac{R(s) + \delta V_u(T)}{\delta + \rho}$ . Since we have assumed that  $V_u(s)$  and  $V'_u(s)$  are continuous and differentiable, the same must also apply to  $R(s)$  and  $R'(s)$ . This smoothness is pivotal for our solution strategy, presented in the next section.

As the following lemma establishes, the reservation wage of workers whose benefit has expired must always be included in the support of the wage offer distribution. Because of the random arrival of job offers, there is always a positive measure of unemployed workers at  $s = 0$  who were unable to find an acceptable job before expiration. If no firms were to target these workers, it would be profitable for some firm to deviate. There would be fewer workers who accept a lower wage offer, but the firm's realized profit after acceptance would be larger; and the latter effect dominates.

**Lemma 1.** *The support of  $F$  must always include  $R(0)$ , unless  $F$  is degenerate at  $R(T)$ .*

In the following section, we present two equilibria that arise from this model. Which occurs depends on how large the expected present value of unemployment benefits,  $b$ , is compared to the net social value of employment,  $p - x$ . We can characterize these equilibria based on how offered wages are targeted to potential employees. If a firm offers wage  $R(s)$ , it is effectively targeting the  $uH(s)$  unemployed workers with  $s$  or less time until benefit expiration.

For instance, when unemployment benefits are very generous, firms only target those whose benefits have expired. In this equilibrium, all firms offer the wage  $R(0)$ , and workers will refuse that wage until their benefits have expired. We refer to this as the *degenerate equilibrium*.

With moderate benefits, firms offer wages  $R(s)$  that are individually targeted for workers across a spectrum of possible remaining benefit durations. Firms are indifferent among these wage offers because a lower wage is only acceptable to a smaller portion of the population, and in equilibrium, the two effects cancel to maintain constant expected profits within the support of  $F$ .

In this *dispersed equilibrium*, however, not all reservation wages are included in the support. The distribution  $F$  only places weight on wages from  $R(0)$  to  $R(S^*)$ , where

$S^* \in (0, T]$ . In other words, workers who have been unemployed fewer than  $T - S^*$  periods (*i.e.* with more than  $S^*$  time until expiration) reject all the equilibrium wage offers, and acceptances only occur later in the unemployment spell.

### 3 Equilibrium Solution

Our approach to finding the equilibrium solution is essentially to reformulate the equilibrium requirements into a second-order differential equation.<sup>7</sup> In particular, we use the first equilibrium requirement to translate the Bellman equations into a relationship between  $R(s)$ ,  $R'(s)$ ,  $R''(s)$ , and  $F(R(s))$ . Meanwhile, the second and third equilibrium requirements are used to translate the steady state conditions into a distinct relationship between these same functions. Together, these allow us to substitute for  $F(R(s))$  and solve the remaining differential equation for  $R(s)$ ; Appendix A.2 demonstrates this process.

This system imposes four boundary conditions. Of course, in the solution to a second-order differential equation, two constants of integration can be chosen to satisfy two of our boundary conditions. A third boundary condition simply determines the equilibrium unemployment rate. The fourth boundary condition determines which reservation wages  $R(s)$  are actually offered by firms in equilibrium, pinning down  $S^*$ .

In the dispersed equilibrium, this last boundary condition produces the following equation, which implicitly solves for  $S^*$  by requiring  $\phi(S^*) = 0$ .<sup>8</sup>

$$\phi(S) \equiv \frac{p - x - b}{b} e^{\frac{2\lambda}{\rho} \left(1 - e^{-\frac{Sp}{2}}\right)} + \frac{\rho + \delta (1 - e^{(S-T)\rho})}{\rho} e^{-\frac{Sp}{2}} - \frac{\delta + \rho}{\lambda}. \quad (11)$$

Intuitively,  $b \cdot \phi(S)$  weighs the net social value of working,  $p - x$ , versus a particular discounted value of unemployment benefits  $b$  (which reflects the probabilities of getting an acceptable offer over the unemployment spell).

If  $S^*$  lies between 0 and  $T$ , what follows will constitute a dispersed equilibrium. If  $\phi(0) \leq 0$ , then setting  $S^* = 0$  in what follows will constitute a degenerate equilibrium.

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<sup>7</sup>van den Berg (1990) uses a differential equation to characterize a partial equilibrium search model. He takes the wage offer distribution as exogenously given, but allows for unemployment benefits, offer arrival rate, and the wage offer distribution to vary deterministically over the unemployment spell.

<sup>8</sup>Equation 11 is obtained from Equation 34 in Appendix A.2, after substituting for  $R(S^*)$  and  $R'(S^*)$  with their solution in Equation 13. This ensures that  $V_u$  is continuous at  $S^*$ .

If  $\phi(T) > 0$ , then an equilibrium does not exist; however, Assumption 2 precisely imposes  $\phi(T) \leq 0$ . The following proposition establishes that equilibrium exists and is unique.

**Lemma 2.** *Under Assumptions 1 and 2, either  $\phi(0) \leq 0$ , or there exists a unique  $S^* \in (0, T]$  such that  $\phi(S^*) = 0$ , but not both.*

Since the two equilibria are mutually exclusive, a degenerate equilibrium occurs if and only if  $\phi(0) \leq 0$ , which is equivalent to saying:

$$p - x \leq b \left( \frac{\delta + \rho}{\lambda} - \frac{\delta}{\rho} (1 - e^{-T\rho}) \right), \quad (12)$$

and is much stronger than Assumption 2. Again, this compares the net social value of employment to a discounted value of the expected flow of unemployment benefits. If benefits are highly generous, households find it optimal to enjoy the full stream of benefits, accepting jobs only after benefits expire. Moreover, firms find it optimal to only offer the wage that is accepted by those without benefits, as the reservation wages of workers with benefits are too high relative to the marginal increase in the number who would accept such an offer.

We now present the equilibrium solution, whether for the degenerate ( $S^* = 0$ ) or dispersed ( $S^* \in (0, T]$ ) equilibrium. We begin by characterizing the reservation wages. Newly unemployed workers will reject all wage offers initially, only accepting some of the offered wages once  $S^*$  time remains until expiration. As a worker nears the expiration of unemployment benefits, he becomes gradually less selective about job offers. After expiration, he accepts any wage in the support of  $F$ .

$$R(s) = \begin{cases} p - \frac{(p-b-x)(\delta+\rho) \left( \rho e^{\frac{S^*\rho}{2}} - \lambda (1 - e^{(S^*-s)\rho}) \right)}{\delta \lambda e^{(S^*-T)\rho} + (\delta+\rho) \left( \rho e^{\frac{S^*\rho}{2}} - \lambda \right)} & s \in (S^*, T] \\ p - \frac{(\delta+\rho)b}{\lambda} e^{-\frac{2\lambda}{\rho} \left( 1 - e^{-\frac{s\rho}{2}} \right)} & s \in [0, S^*]. \end{cases} \quad (13)$$

In equilibrium, the reservation wages for  $s > S^*$  are not offered by any firm; even so, these represent the hypothetical wages at which the unemployed would be indifferent between entering employment rather than continuing unemployment, taking  $F$  as given. Note that  $R(s)$  is continuous at  $s = S^*$ ; to show this, one must substitute for  $p - b - x$  using  $\phi(S^*) = 0$ .

Next, we consider the distribution of offered wages:

$$F(w) = \begin{cases} 0 & w < R(0) \\ \frac{\rho}{2\lambda} \left( 1 - \ln \left( \frac{\lambda(p-w)}{(\delta+\rho)b} \right) \right) & w \in [R(0), R(S^*)] \\ 1 & w \geq R(S^*). \end{cases} \quad (14)$$

It is also convenient to express this distribution in terms of  $s$ , which is to say, what fraction of offers are at or below the reservation wage of a person with  $s$  time remaining until benefit expiration. This is done by substituting  $w = R(s)$  into  $F(w)$ .

$$F(R(s)) = \begin{cases} 1 & s \in [S^*, T] \\ \frac{\rho}{2\lambda} + 1 - e^{-\frac{s\rho}{2}} & s \in [0, S^*). \end{cases} \quad (15)$$

Notice that there is an atom of  $\frac{\rho}{2\lambda}$  at  $s = 0$  (or the lowest offered wage), and an atom of  $e^{-\frac{S^*\rho}{2}} - \frac{\rho}{2\lambda}$  at  $s = S^*$  (or the highest offered wage), with an atomless distribution spanning in between. The atomless portion of the distribution has increasing density in  $w$ , so higher wages are more likely than low wages. This is typically the case in search models with endogenous wages and homogeneous productivity; the difference here is that the atom at  $w_\ell$  can skew the distribution back towards lower wages. Also, take note that Assumption 1 is needed here to ensure that  $F(R(s)) \leq 1$  for all  $s$ , as required for a well-defined c.d.f.

With these  $R$  and  $F$ , the distribution of the unemployed becomes:

$$H(s) = \begin{cases} \frac{2\lambda \left( e^{-\frac{S^*\rho}{2}} - e^{-\frac{s\rho}{2}} \right)}{e^{-\frac{S^*\rho}{2}} - \frac{\rho}{2\lambda}} & s \leq S^* \\ \frac{1 + \lambda(T - S^*)e^{-\frac{S^*\rho}{2}}}{1 + \lambda(s - S^*)e^{-\frac{s\rho}{2}}} & s > S^*. \end{cases} \quad (16)$$

This distribution is continuous for all  $s \in [0, T]$ , including at  $S^*$ , and leaves an atom at  $H(0)$ . The distribution of the employed is:

$$G(w) = \begin{cases} 0 & w < R(0) \\ \frac{\rho(\delta+\rho)b}{2(p-w)\lambda^2} e^{\frac{S^*\rho}{2} - \frac{2\lambda}{\rho}} \left( 1 - e^{-\frac{S^*\rho}{2}} \right) & w \in [R(0), R(S^*)] \\ 1 & w \geq R(S^*). \end{cases} \quad (17)$$

Atoms also occur in  $G$  at the highest and lowest wages, though the former is larger and the latter smaller than in  $F$ .

Steady state profits are:

$$\pi = \frac{(\delta + \rho)b}{\lambda} e^{\frac{S^*\rho}{2} - \frac{2\lambda}{\rho} \left(1 - e^{-\frac{S^*\rho}{2}}\right)}, \quad (18)$$

and the unemployment rate is:

$$u = 1 - \frac{\lambda}{\lambda(1 + \delta(T - S^*)) + \delta e^{\frac{S^*\rho}{2}}}. \quad (19)$$

**Proposition 1.** *Assume that either  $\phi(S^*) = 0$ , or  $\phi(0) < 0$  and  $S^* = 0$ . Then Equations 13 through 19 constitute an equilibrium.*

One curious feature of this equilibrium is the two atoms in the wage offer distribution. On the one hand, since a positive mass of unemployed workers have lost their unemployment benefits, it is not surprising that a mass of firms would target them. But the same *cannot* be said of the atom at  $R(S^*)$ ; what explains this phenomenon?

This atom is essentially a residual. The shape of  $R(s)$  and  $F(R(s))$  are carefully pinned down for  $s \in [0, S^*)$  by the Bellman equations (governing the rate at which reservation wages fall) and the steady state conditions (governing how many workers remain in state  $s$ ), which must interact to keep expected profits equal.  $R(S^*)$  is then pinned down in order to keep  $V_u$  continuous at  $S^*$ , but  $F(R(S^*))$  need not be — note that in Equation 33 the atom  $\mu_F(w_h)$  drops out of the Bellman equation at  $s = S^*$ . Instead,  $F(R(S^*))$  must equal 1, since  $R(S^*)$  is the highest wage offered.<sup>9</sup>

Using this solution, we compute typical labor statistics of interest. Of course, one of the most fundamental, the unemployment rate, was already reported in Equation 19. Next, consider descriptive statistics about the distribution of wages among current employees.

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<sup>9</sup>In an on-the-job search environment, firms would avoid offering a wage that a mass of other firms offer. A slightly higher wage causes a discrete jump in the fraction of workers who are willing to accept it — the positive mass of employed workers at that wage now strictly prefer the offer. Thus no atoms occur in the pure wage dispersion model of Burdett and Mortensen (1998). In our model this sort of direct competition does not occur. When a firm encounters a worker, its offer is the only wage under consideration. Thus, the firm offering  $R(S^*)$  only considers the probability that the worker has  $S^*$  or less time until expiration, not the measure of other firms offering  $R(S^*)$ .

$$w_{\max} = p - \frac{(\delta + \rho)b}{\lambda} e^{-\frac{2\lambda}{\rho}} \left(1 - e^{-\frac{S^*\rho}{2}}\right) \quad (20)$$

$$w_{\text{mean}} = p - \frac{(\delta + \rho)b}{\lambda} e^{\frac{S^*\rho}{2} - \frac{2\lambda}{\rho}} \left(1 - e^{-\frac{S^*\rho}{2}}\right) \quad (21)$$

$$w_{\text{median}} = p - \frac{\rho(\delta + \rho)b}{\lambda^2} e^{\frac{S^*\rho}{2} - \frac{2\lambda}{\rho}} \left(1 - e^{-\frac{S^*\rho}{2}}\right) \quad (22)$$

$$w_{\min} = p - \frac{(\delta + \rho)b}{\lambda} \quad (23)$$

Another wage implication for the model is that firms using a high wage strategy will, on average, employ more workers than those with a low wage strategy. The steady state number of employees at a given wage per firm offering that wage is  $\frac{g(w)}{f(w)} = \frac{p - w_{\text{mean}}}{p - w}$ , which is increasing in  $w$ . Thus, firm size is positively correlated with wages, a well-documented empirical fact.

The length of unemployment spells is another obvious point of interest. Here we provide two measures. The first is expected duration,  $E[d]$ , for a newly unemployed worker. This computes to:

$$E[d] = T - S^* + \frac{e^{\frac{S^*\rho}{2}}}{\lambda}. \quad (24)$$

The second measure is  $H(0)$ , the fraction of unemployed workers who have exhausted their unemployment benefits. This gives us a measure of lengthy unemployment spells.

$$H(0) = \frac{e^{-\frac{2\lambda}{\rho}} \left(1 - e^{-\frac{S^*\rho}{2}}\right)}{1 + \lambda(T - S^*)e^{-\frac{S^*\rho}{2}}}. \quad (25)$$

We also use this measure in computing the cost of providing unemployment benefits. The total number of workers still covered by unemployment benefits at any given moment is  $u(1 - H(0))$ ; if multiplied by  $b$ , this gives the cost of unemployment benefits over one unit of time.

A frequently studied statistic is the hazard rate of unemployment exit, which is the rate at which workers find acceptable jobs. For a worker who has  $s$  time until benefit expiration, this is:

$$\eta(s) \equiv \lambda(1 - F(R(s))) = \lambda e^{-\frac{s\rho}{2}} - \frac{\rho}{2}. \quad (26)$$



At  $s = 0$ , the hazard rate discretely jumps to  $\lambda$ , due to the atom at  $R(0)$ . The average hazard rate over all unemployed workers is:

$$\bar{\eta} \equiv \frac{\lambda}{(T - S^*)\lambda + e^{\frac{S^*p}{2}}}. \quad (27)$$

Finally, if we define total welfare in this economy as the average utility of workers plus firm profits minus the cost of providing unemployment benefits, the result is  $(1 - u)p + ux$ . Since we assume that  $p > x$ , any policy that reduces unemployment is welfare enhancing. Of course, this critically relies on the assumption that workers are risk neutral; risk aversion would introduce a welfare-enhancing insurance role for unemployment benefits.

## 4 Comparative Statics

We now consider several policy experiments and determine their impact on the economy through comparative statics. In doing so, of course, we are comparing steady state outcomes, as the dynamic transition is not modeled. Because our equilibrium is implicitly solved in terms of  $S^*$ , we use implicit differentiation to determine how this responds to changes in  $b$  or  $T$ , and the signs are unambiguous. We provide intuition for these results in the subsections that follow.

**Lemma 3.** *Under Assumption 1,  $\frac{\partial S^*}{\partial b} < 0$  and  $\frac{\partial S^*}{\partial T} > 0$  in a dispersed equilibrium.*

Using the calculated implicit derivatives, we determine the impact of any particular policy change on labor market statistics. We concentrate on the dispersed equilibria, though most of the signs remain the same in a degenerate equilibrium.

### 4.1 Policy Experiment: Increase in $b$

Suppose that benefits are made more generous, keeping the duration unchanged. Table 1 summarizes the comparative statics for the key statistics computed in Section 3. Notice that the entire wage distribution shifts downward after a benefit increase. One might intuitively expect that a larger benefit creates a higher opportunity cost of accepting an offer, so firms must respond by offering higher wages. Indeed, this occurs in the standard on-the-job-search model of Burdett and Mortensen (1998). It is all the more surprising, then, that exactly the opposite occurs in our equilibrium.

Table 1: Comparative Statics in a Dispersed Equilibrium

	$\partial/\partial b$	$\partial/\partial x$	$\partial/\partial T$	$\partial/\partial \delta$
$w_{\max}$	–	–	+	–
$w_{\text{mean}}, w_{\text{median}}$	–	–	+	–
$w_{\min}$	–	0	0	–
$u$	+	+	$\left(1 + \frac{\rho}{\lambda} e^{\frac{S^* \rho}{2}}\right) e^{(S^* - T)\rho} < \frac{\delta + \rho}{\delta}$	+
$E[d]$	+	+	$\left(1 + \frac{\rho}{\lambda} e^{\frac{S^* \rho}{2}}\right) e^{(S^* - T)\rho} < \frac{\delta + \rho}{\delta}$	$1 - e^{(S^* - T)\rho} < \frac{\rho}{\lambda} e^{\frac{S^* \rho}{2}}$
$H(0)$	+	+	–	$1 - e^{(S^* - T)\rho} < \frac{\rho}{\lambda} e^{\frac{S^* \rho}{2}}$
$\pi$	+	+	–	+

*Notes:* Where a condition is listed, it means the comparative static is strictly positive if and only if this condition is satisfied.

What is the source of this counterintuitive result? As noted in Lemma 3,  $\frac{\partial S^*}{\partial b} < 0$ . Thus the more generous benefit encourages unemployed workers to wait until *later* in their unemployment spell before accepting any wage offer.

However, this blessing of increased patience becomes its own curse as benefit expiration nears. With a higher  $b$ ,  $R'(0)$  is bigger (as seen in Eq. 32, in Appendix A.2), which means the reservation wage falls at a faster rate — workers are more anxious to secure a job before benefits are lost. In addition, there are more workers near expiration than before, which is visible in the fact that  $H(0)$  increases. Both of these effects allow firms to offer lower wages than before and yet be more likely to fill the positions. Note that Equations 32 and 37 give us  $b(\delta + \rho) = \lambda(p - R(0))$ ; thus as  $b$  increases,  $R(0)$  must fall.

The key driving force of this result is the sharp drop in benefits at  $s = 0$ . The delay induced by an increase in  $b$  results in more workers accumulating near (and over) the edge of that cliff, all of whom are more willing to accept low wage offers than before. At the same time, workers early in their unemployment spell are actually more reluctant to accept any offers.<sup>10</sup>

An alternative explanation for decreasing wages is the so-called entitlement ef-

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<sup>10</sup>In Albrecht and Vroman (2005), an increase in  $b$  decreases both the high and low wages, but it also *decreases* the fraction of unemployed workers whose benefits expire. This occurs because their Poisson transition to expiration aggregates all workers who still have benefits, and thus cannot distinguish the effect on those early versus late in their unemployment spell.

fect.<sup>11</sup> This states that when benefits are more generous, workers near expiration are willing to accept lower wages so as to renew eligibility for the UI program. In our equilibrium, it is true that a newly unemployed worker has a discounted flow of utility  $V_u(T)$  that is larger than even  $V_e(w_{\max})$ , a worker's discounted flow of utility when employed at the highest wage,<sup>12</sup> so this could potentially influence a worker's decisions.

On further inspection, however, the entitlement effect appears to play a minor role in this model. To test this, we reformulated the model so that once a worker accepts a job, it is held until death, which randomly arrives at rate  $\delta$ . New workers enter the model at rate  $\delta$ , being unemployed with  $T$  periods of benefits.<sup>13</sup> This effectively shuts down any entitlement effect, since workers will never become unemployed again.

Surprisingly, the solution is virtually unaltered; the only significant change is to the equilibrium condition analogous to  $\phi(S^*) = 0$ . In particular, the reservation wage and hazard rate for  $s \in [0, S^*]$  are unchanged, as are their comparative statics with respect to  $b$ . Thus, wages fall, not because workers are anxious to renew eligibility, but because firms exploit the impending benefit loss of a larger fraction of workers.

The cost of providing these benefits,  $u(1 - H(0))b$ , has competing effects since the second term falls while the first and third rise, but on net, the cost unambiguously rises. Not surprisingly, an increase in  $x$  has similar effects in almost all aspects.

It is worth noting that as  $b$  increases, a dispersed equilibrium will move closer to becoming a degenerate equilibrium. This is not surprising, since the higher benefit encourages delay. Eventually, firms will stop targeting workers who are still receiving benefits, since the higher wages needed to attract them will not be worth the marginal increase in probability of being accepted.

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<sup>11</sup>This effect is theoretically discussed in Mortensen (1977) and empirically investigated in Katz and Meyer (1990) and Lalive, *et al* (2006), in the context of hazard rates of employment.

<sup>12</sup>Our model does not allow voluntary quits, which is a sensible restriction if quitters are ineligible for benefits (as is common in most unemployment insurance programs). If so, a worker who quits would transition to  $V_u(0)$ , which is never strictly preferred to working. Albrecht and Vroman (2005) provide an extension in which quits are only identified with some exogenous probability; this raises wages and discourages high wage workers from quitting. We believe the same would occur here; also, we suspect this would ensure that the full distribution of wages are offered, from  $R(0)$  to  $R(T)$ .

<sup>13</sup>Coles and Masters (2004) employ a similar environment.

## 4.2 Policy Experiment: Increase in $T$

The other typical policy tool is to extend benefits over a longer time span; the results of this policy change are also reported in Table 1. In a dispersed equilibrium, an increase in  $T$  results in higher (and more dispersed) wages. While increases in  $b$  encourage delay followed by a hasty exit, increases in  $T$  actually stretch out the exit, making workers less desperate and forcing firms to offer higher wages. Note that fewer unemployed workers exhaust their benefits, since  $H(0)$  falls after the policy change.<sup>14</sup>

To be more particular,  $\frac{\partial S^*}{\partial T} > 0$ , so workers begin accepting wages earlier after the change. Of course, this is relative to the date at which benefits expire, which has moved farther away. To have unemployed workers accepting offers earlier after the change, relative to the *beginning* of their unemployment spell, we would need  $\frac{\partial S^*}{\partial T} > 1$ , which may or may not hold. To the extent that  $\frac{\partial S^*}{\partial T} < 1$ , we might view this as one measure of moral hazard, since the longer unemployment benefit induces a longer wait before accepting offers.

Regardless of this moral hazard, the net effect is unambiguous in raising wages, and a larger fraction of the unemployed find jobs before losing eligibility. The effect on unemployment rates and average unemployment duration can go either way, but will typically rise whenever  $\frac{\partial S^*}{\partial T} < 1$ .

The degenerate equilibrium, however, is quite different from the dispersed equilibrium. An increase in  $T$  actually reduces the (only) wage offered. To explain this, note that  $V_u(0) = \frac{x}{\rho}$  in equilibrium. In the degenerate equilibrium, firms offer a wage which makes these expired workers indifferent,  $V_u(0) = \frac{R(0) + \delta V_u(T)}{\delta + \rho}$ , but the longer benefit increases the present value of starting unemployment,  $V_u(T)$ . Hence, the firms expropriate the unemployment benefit by offering a lower wage than before.

Concerning transitions between equilibria, as  $T$  increases, Equation 12 is less likely to be satisfied. Thus, beginning with a degenerate equilibrium, an increase in  $T$  will eventually transition to a dispersed equilibrium, whose support continues to grow as  $T$  increases. Hence, countries with very long potential benefit durations are most likely to find themselves in a dispersed equilibrium.

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<sup>14</sup>This contrasts with Albrecht and Vroman (2005). There, a decrease in the probability of losing benefits (the analog of increasing  $T$ ) causes no change in the high wage and a decrease in the low wage; so on average, wages fall.

### 4.3 Response to Shock: Increase in $\delta$

We now ask what occurs if the exogenous job separation rate were to permanently increase. Of course, the model is only solved for steady state, so this represents the effect after sufficient time for transition; but one might think of this as a deterioration of macroeconomic conditions or increased friction due to other labor market policies.

Not surprisingly, the greater flow of workers into unemployment results in a higher unemployment rate and shifts the entire wage distribution downward. Since employment spells will be shorter on average now, this shock reduces the value of being employed; in that sense, it has similar impact as raising the value of being unemployed. Firms have a larger pool of unemployed workers to draw from, and workers are willing to accept lower wages.

Curiously, the increase in  $\delta$  does not immediately imply an increase in average duration or in the number of workers whose benefits expire. These increase if and only if  $S^*$  decreases, and this depends on parameter values but is more likely to occur when  $S^*$  is near  $T$ .

These comparative statics may explain why unemployment benefits are often extended but not increased during recessions. *Longer* benefit periods can counteract the downward pressure on wages created by increased job destruction, while *higher* benefits would exacerbate the wage decrease.

## 5 Empirical Evidence

Many empirical studies of unemployment insurance have recognized the importance of limited benefits in worker behavior, including Katz and Meyer (1990), van Ours and Vodopivec (2006), Lalive, *et al* (2006), Card, Chetty and Weber (2007), and Lalive (2007, 2008). In this section, we summarize these results and compare them to the predictions of our model. This empirical literature focuses on how hazard rates and expected unemployment duration respond to policy changes. On a qualitative level, our model replicates nearly all of the effects found in the data, not only for changes in average hazard rates, but also for the response of hazard rates at a given time in the unemployment spell.

To allow some quantitative comparisons, we begin by calibrating the model to

match a few stylized facts about the US labor market, summarized in Table 2.<sup>15</sup> These targets are computed from CPS data on workers with no more than high school education, averaged across 2000 - 2010.<sup>16</sup>

Table 2: Calibration Targets

Parameter	Discount Rate		Target	Observed
	Low	High		
$T$	6	6	Potential Benefit Duration ( $T$ )	6 months
$\rho$	0.0041	0.041	Annual interest rate (for low $\rho$ )	5.0%
$\delta$	0.022	0.022	Unemployment Rate ( $u$ )	7.15%
$\lambda$	0.311	0.328	Hazard Rate ( $\bar{\eta}$ )	0.29
$b$	0.496	0.491	Replacement Rate ( $b/w_{\text{mean}}$ )	50 %
$x$	0.580	0.563	Benefit Exhaustion Rate ( $H(0)$ )	15.7%

*Note:* The last four parameters are uniquely determined by jointly solving for the last four targets.

We consider one unit of time to equal one month. Marginal product of a worker is normalized to  $p = 1$ ; this choice proportionately scales  $b$ ,  $x$ , and  $R(s)$ , but otherwise has no impact on equilibrium outcomes. To determine our six parameters, we need one additional target: the annual interest rate. To obtain the typical 5% rate, one would need  $\rho = 0.0041$ . It is reasonable, though, that the effective interest rate for unemployed low-skilled workers could be much higher due to credit constraints and negligible savings. To provide an extreme example, we provide an alternative calibration for  $\rho = 0.041$ . This higher discount rate creates greater wage dispersion

<sup>15</sup>The rates of job arrival, destruction, and the benefit replacement are close to those documented in Shimer (2005) for the aggregate US labor market, even though we focus on a particular education level in our calculations.

<sup>16</sup>The CPS lacks sufficient information to compute the replacement rate,  $b/w_{\text{mean}}$ ; rather, this is chosen to match the institutional details. The overwhelming majority of states provide benefits equal to half of the worker's former wage; some are as high as 60%. This is subject to a maximum benefit, but since this group of workers earn \$23,000 on average, their benefit falls well below the maximum benefit. Regarding the fraction of workers who exhaust their benefits, the CPS does not provide enough detail to directly compute this; instead, we compute the fraction of unemployed workers whose unemployment spell exceeds 26 weeks. Not all unemployed workers are eligible for unemployment, but we improve our estimate by restricting the sample to those who lost their job (rather than workers who quit their job or have (re-)entered the labor force, and thus would have been ineligible). Eligibility also depends on past earnings, but these requirements are not very stringent. Our estimate is only slightly lower than what Katz and Meyer (1990) estimated for the full labor market over the 1980s.

and mimics the effect of risk aversion when log preferences are employed, as discussed in Section 6.1.

Under either set of parameters, Assumptions 1 and 2 are easily satisfied, and we obtain a dispersed equilibrium, with  $S^* = 5.75$  for the low discount rate and  $S^* = 5.94$  for the high discount rate. Note that since  $S^*$  is close to  $T$ , workers are willing to accept some jobs almost immediately on entering unemployment. The resulting wage offer distribution is depicted in Figure 1.

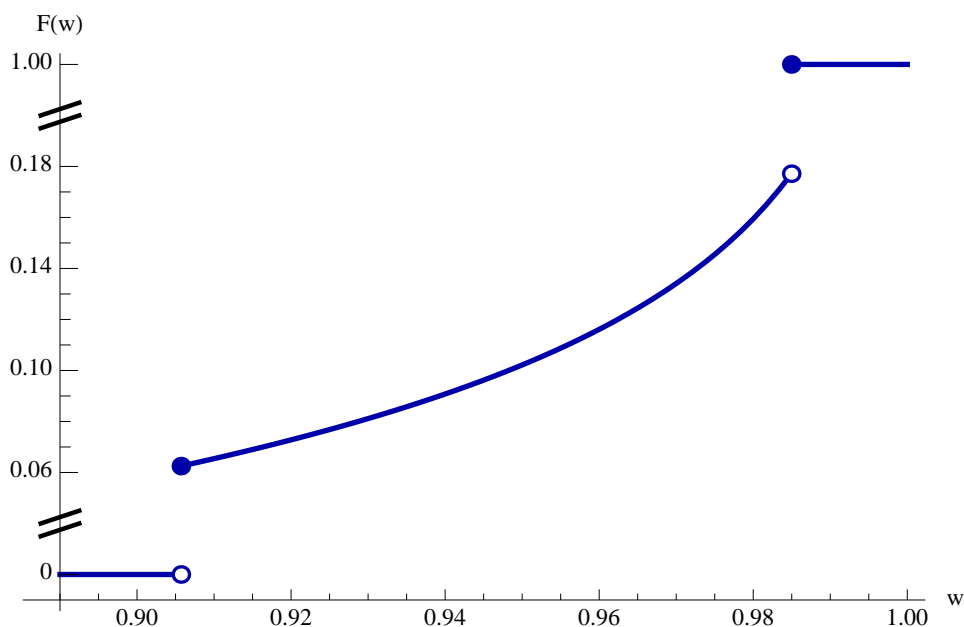


Figure 1: The cumulative distribution,  $F(w)$ , of wage offers,  $w$ , for  $\rho = 0.041$ , relative to marginal product  $p = 1$ .

## 5.1 Hazard Rates

The hazard rate of exiting unemployment has been the focus of much research. Katz and Meyer (1990) and Meyer (1990) specifically investigated the impact of limited benefit duration on hazard rates. They found that hazard rates gradually rise as benefit exhaustion approaches, with a significant jump (78% increase) in the last week of benefits. Even so, only a small fraction of unemployment spells end near the benefit expiration date. Similar results have been found in a variety of studies, such

as Addison and Portugal (2004).<sup>17</sup>

Katz and Meyer (1990) cite the partial equilibrium model of Mortensen (1977), which takes the wage offer distribution as given, as the theoretical basis for their empirical findings. This model does predict a steady increase in the hazard rate up until benefit exhaustion, but a sudden jump occurs at expiration only if income and leisure are strict complements. In an empirical study of hazard rates, Lalive, *et al* (2006) note that aside from this, “there is no theoretical explanation for the existence of end-of-benefit spikes.”

Our model can offer an explanation under milder assumptions. A positive measure of firms always offer the lowest wage, which is acceptable only to workers whose benefits have expired. Thus, the hazard rate jumps from  $\lambda - \frac{\rho}{2}$  to  $\lambda$ ; for our calibrations, that produces a 1% jump (or 6.6% jump for high  $\rho$ ), illustrated in Figure 2. This atom in the wage offer distribution is a necessary condition to solve our differential equation of  $R(s)$ ; without it, either the Bellman equation  $V_u(s)$  would be discontinuous at  $s = 0$ , or the flow of workers with expired benefits (Eq. 8) would not remain in steady state. Note that, in addition to the jump at expiration, our model also predicts rising hazard rates throughout the benefit period. Moreover, in our calibration, the jump at  $s = 0$  is nearly as large as the continuous rise over the preceding 6 months.

One fact that our model cannot explain is that hazard rates gradually decline *after* benefit expiration. It could be that employers see long unemployment spells as a bad signal of worker quality; to the extent that a firm could imprecisely observe long gaps in employment histories, they would reduce either  $\lambda$  or  $w$ . Also, our model predicts a hazard rate of 0 for the first  $T - S^*$  periods of an unemployment spell; while hazard rates begin low, they are certainly not literally zero. Of course, this could just mean that  $S^*$  is close to  $T$ , as it is in our calibration.

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<sup>17</sup>The result is significantly weaker in Card, Chetty and Weber (2007). They find that the jump mostly occurs because people stop registering in the unemployment system once they lose benefits, not because they become more likely to find an acceptable job. After distinguishing between these two outcomes, they still find a sudden increase in the hazard rate of exiting unemployment to employment, but it is only a 12 to 14% jump. In addition, not many workers are affected by this jump, with less than 1% of unemployment spells ending near benefit expiration.

The hazard rate also increases over the benefit period, but the standard errors are large enough that one cannot reject constant hazard rates. Using detailed administrative data, van Ours and Vodopivec (2006) also distinguish between stopping registering and finding a job in Slovenia; there the spike is nearly as large as in Katz and Meyer (1990).



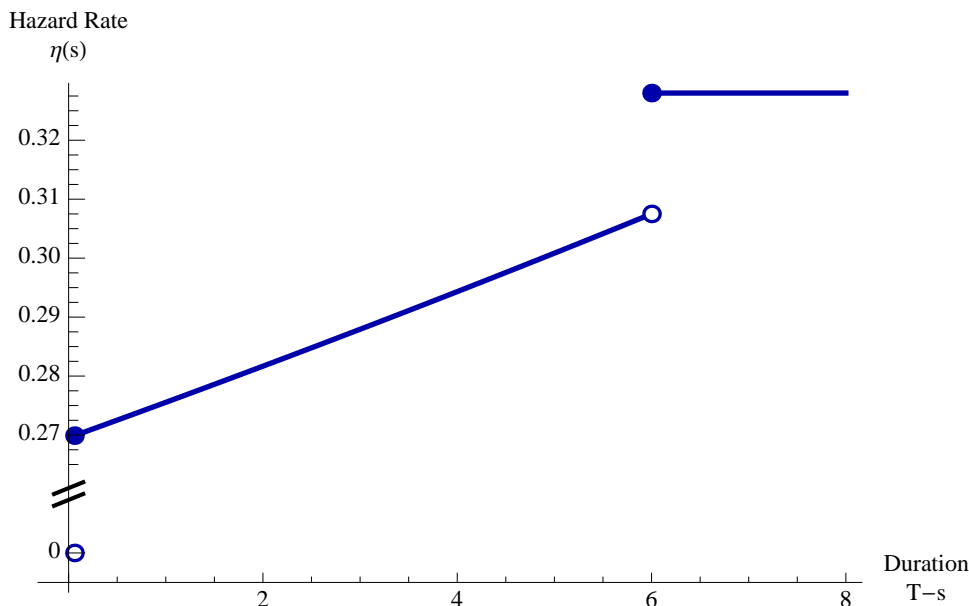


Figure 2: The hazard rate of exiting unemployment,  $\nu(s)$ , for a worker unemployed for  $T - s$  periods, for  $\rho = 0.041$ .

## 5.2 Comparative Statics on $b$

When unemployment benefits are increased, our model predicts several patterns in the equilibrium response. First, average hazard rates fall: in our calibration, a 10% increase in benefits leads to a 25.7% decrease (or 29.5% for high  $\rho$ ) in the average hazard rate. However, this is entirely due to changed behavior of workers with approximately  $S^*$  time until benefit expiration. All others still exit unemployment at the same hazard rate as before.<sup>18</sup>

This pattern is confirmed in the data. Using US data, Katz and Meyer (1990) find that a 10% increase in the level of benefits leads to a 5.4% decrease in the average hazard rate. But this effect is concentrated primarily among those with a longer span of remaining benefits; those within three weeks of benefit exhaustion have a statistically insignificant increase in their hazard rate. Lalive, *et al* (2006) find a similar pattern on hazard rates: lower for those early in an unemployment spell and no significant change for those near or beyond benefit exhaustion.

<sup>18</sup>Note that Mortensen (1977) predicted that an increase in  $b$  would reduce hazard rates of workers early in an unemployment spell but increase hazard rates for those late in the spell. The same was predicted for increases in  $T$ .

The other prediction of the model is that wages will fall: in our calibration, a 10% increase in  $b$  will cause the average wage to fall by 0.4% (or 1.2% for high  $\rho$ ), with similar effect on the maximum, median, and minimum wages. There are fewer studies on this topic. Blau and Robins (1986) find that more generous benefits do not create a statistically significant change in offered wages, though the point estimates are positive. They also offer point estimates that a 10% increase in benefits raises reservation wages by 2.1% for men and 1.5% for women, but could not compute standard errors. Ehrenberg and Oaxaca (1976) distinguish workers by gender and age in their analysis. Older men see a statistically significant 2.8% increase in wages after a 10% increase in benefits, but young men (under 25) have a statistically insignificant point estimate just above zero.

Our model is making equilibrium predictions, so to the extent that Blau and Robins (1986) and Ehrenberg and Oaxaca (1976) rely on individual variation that is unobservable to the firm (as opposed to variation between state programs which creates distinct labor markets), these would not be an adequate test of our model. Even so, we are not surprised that older workers might be able to leverage a higher benefit to their advantage while younger workers cannot. Those with greater experience are more likely to bargain over their wages; therefore, our wage posting environment would not be as applicable.

### 5.3 Comparative Statics on $T$

Our model also predicts a distinct pattern of hazard rate changes when unemployment benefits are extended. Again, the average rate falls dramatically: in our calibration, a 50% increase in potential benefit duration (26 to 39 weeks) will cause a 44% drop in hazard rates (and almost the same for high  $\rho$ ). As with  $b$ , this effect is concentrated on those workers with approximately  $S^*$  time remaining until benefit expiration, since the hazard rate for a worker with  $s$  time until expiration is not dependent on  $T$  except near  $S^*$ .

Even so, most empirical studies calculate hazard rates for a given unemployment duration, rather than a given time until benefit expiration. On this basis, our model would predict no change for workers with long spells (*i.e.* unemployed longer than the new  $T_1$ ) or those early in their unemployment spell (unemployed between 0 and the original  $T_0 - S_0^*$ ). If  $\frac{\partial S^*}{\partial T} < 1$ , as it is with our calibration, then all workers with

intermediate spells will experience lower hazard rates, with the strongest decrease just above  $T_0 - S_0^*$ .

These patterns are again reflected in the data. Katz and Meyer (1990) conclude that a 50% increase in potential benefit duration (PBD) would cause a 27% decrease in the average hazard rate, a 12% increase in average duration, and a 60% drop in the fraction of unemployed workers who exhaust their benefits. Lalive (2007, 2008) arrives at similar predictions for the average unemployment duration among men and even stronger results for women.

van Ours and Vodopivec (2006) provides more detail about hazard rates over unemployment spells. Using Slovenia's reduction in potential benefit duration as a natural experiment, they trace out the hazard rate at three month intervals. For those whose PBD was reduced from 18 months to 9 months, there was virtually no change in the 0-6 month ranges, but a significant increase in the 6-12 month ranges. A similar pattern emerged for those whose PBD was reduced from 12 months to 6 months.

Lalive, *et al* (2006) also investigates the opposite natural experiment, where a policy change granted Austrian workers with longer employment spells an additional 9 weeks of benefits beyond the standard 30 weeks. Between weeks 16 and 39 of their unemployment spell, these workers saw significantly lower hazard rates compared to those restricted to the standard benefit. However, they also saw a relative increase in their hazard rate between weeks 42 and 50 (after benefits had expired for either type of worker), which cannot be generated by our model. The same pattern occurs among those eligible for a larger 22 week increase, with lower hazard rates for the 35 weeks before exhaustion and higher rates for the 8 weeks following. In either case, the average hazard rate still falls, as predicted by our model: increasing a worker's PBD by 30% leads to a 6.9% decrease in his hazard rate.

Evidence on the effect of PBD on wages is scant. Lalive (2007) reports that after this Austrian experiment, there was no significant change in wages accepted by unemployed workers. Our model predicts an increase in wages, but the increase is minuscule; for our calibration, a 50% extension only results in a 0.04% increase (or 0.10% for high  $\rho$ ) in the average accepted wage.

## 5.4 Wage Dispersion

A final empirical question is raised by Hornstein, *et al* (2011), which proposed the ratio of mean-to-minimum wage as a useful measure of wage dispersion. In our low  $\rho$  calibration, that ratio would be:

$$Mm \equiv \frac{w_{\text{mean}}}{w_{\text{min}}} = 1 + \frac{b(\delta + \rho)}{p\lambda - b(\delta + \rho)} \left( 1 - e^{\frac{T\rho - 2\lambda}{2} \left( 1 - e^{-\frac{T\rho}{2}} \right)} \right) = 1.036. \quad (28)$$

This is well below the 1.68 mean-to-minimum ratio that Hornstein, *et al* (2011) measure in the data; however, their calibration of a basic sequential search model produced this same 1.036 ratio. They also examine many extensions of the benchmark model, concluding that reasonable calibrations would still fall remarkably short. For instance, the on-the-job search model that serves as the basis for most work on equilibrium wage dispersion yields  $Mm = 1.10$ . Thus, while our model cannot explain all of the observed wage dispersion, its contribution is on the same order of magnitude as well-accepted models.

It may also be possible to obtain more dispersion through extensions to our model, such as risk aversion or endogenous search effort, as these were helpful, if not fully successful, in Hornstein, *et al* (2011). As previously mentioned, under the high  $\rho$  calibration, our model behaves very much like it does with log preferences described in Section 6.1, and generates  $Mm = 1.085$ .

## 6 Extensions

We now briefly consider some useful features that can be incorporated in the model. These extensions have minor impact on the qualitative features of our equilibrium, but can affect our comparative statics. They also lead to a more complicated statement of the solutions; for brevity, we do not present the solutions here, but provide a comparison to the dispersed equilibrium in our baseline framework.

### 6.1 Risk Aversion

To this point, we have assumed that utility is linear in consumption, and one might rightfully ask how consequential this assumption is. However, the model ceases to be

analytically tractable for most strictly concave utility functions. The difficulty is in solving the second order differential equation which aligns workers' reservation wages and firms' constant profits across wages. In the case of log preferences, though, this is possible, and the solution shows remarkable resemblance to the baseline outcome.<sup>19</sup>

First, the analog to Equation 11 which determines the cutoff point  $S^*$  is significantly more complicated, but given the same parameters will generally result in a higher cutoff. In other words, workers will find some acceptable wages earlier in their unemployment spell. This seems to drive most of the other differences in equilibrium outcomes. This allows workers to exit unemployment earlier, so fewer workers reach expiration and the unemployment rate is lower.

Indeed, the surprising effect is that the distribution of offered wages is higher than with risk-neutral workers. With fewer unemployed workers, firms compete more over them; in fact, the atom on the highest wage is also larger. Consequently, reservation wages are slightly higher with risk averse workers. Comparative statics must be numerically computed, but these all retain the same sign as in the baseline model.

## 6.2 Taxation

The financing of unemployment benefits is a potentially important institutional detail. Of course, with linear utility the timing of taxation is unimportant; indeed, in the baseline model, one could consider benefits as being financed through lump-sum taxation without any change in the outcomes.

However, a proportional tax may introduce distortions, and is more empirically relevant. In the United States, UI systems are funded by payroll taxation, nominally assigned to the employer.<sup>20</sup> We impose a constant tax rate  $\tau$  on the wages of all employed workers. This reduces a firm's profit to:  $(p - (1 + \tau)w)\frac{f(w)}{g(w)}$ . In equilibrium, wages adjust so that the tax incidence is shared; we reach the same result if taxes are imposed on workers instead of firms.

We require the UI system to maintain a balanced budget; thus, total benefits paid each period must equal total tax revenue:  $u(1 - H(0))b = (1 - u)\tau w_{\text{mean}}$ . Thus, tax rate  $\tau$  is endogenously determined by the exogenous benefit level  $b$  (or vice versa), and the balanced budget is added to our equilibrium conditions. Beyond this, the

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<sup>19</sup>We assume that workers have no savings technology, so they live hand-to-mouth.

<sup>20</sup>Typically this tax is experience-rated, so a firm which dismisses more workers pay a higher rate, but since job destruction is exogenous in our model, we omit this feature.

equilibrium solution is nearly identical except to replace price  $p$  with  $\frac{p}{1+\tau}$  wherever it occurs.

Again, perhaps the most important effect is on Equation 11; given the same benefit level and parameters, introducing taxes will result in a lower  $S^*$ . As a consequence, the unemployment rate increases and more workers have longer unemployment spells. Reservations wages are lower (by  $\frac{\tau}{1+\tau}p$  for all  $s$ ), but the distribution of offered wages  $F(R(s))$  is unchanged (though  $F(w)$  shifts downward).

We evaluate the effects of changes in  $b$  or  $T$  numerically (because we now have two implicit conditions that must be satisfied: the balanced budget constraint and  $\phi(S^*) = 0$ ), and the increase in benefit levels has the same effect as before: a fall in wages, an increase in unemployment, and a rise in after-tax profits. An increase in potential benefit duration has similar effect to an increase in benefit levels; however, this is opposite the predictions in the baseline model. While a longer benefit period allows workers to be more patient and demand higher wages, the increased taxes necessary to sustain the longer benefit will have the opposite effect. In our calibration, the latter effect dominates.

### 6.3 Firm search effort

Since firms make positive profits in the baseline model, it is natural to ask what would happen if new firms were allowed to enter the market. One would naturally expect workers to find it easier to receive job offers when there are more potential employers with whom they might interact. Thus, the job offer rate  $\lambda$  becomes an endogenous variable.

To provide the simplest illustration of firm search effort, we assume each firm incurs a cost  $k$  to operate in this market (regardless of the number of employees). This is compared to the expected steady-state profit  $\pi$ , and we impose a zero profit condition in equilibrium. We then allow  $\lambda$  to adjust so as to satisfy this condition;<sup>21</sup> if profits are higher than  $k$ ,  $\lambda$  would rise, which can be interpreted as firms entering the market.

The equilibrium solution is exactly as presented in the baseline case; the only

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<sup>21</sup>Following the tradition of labor matching models, one could introduce  $\lambda$  as a function of the number of firms and the number of unemployed workers. The basic insights would proceed similarly, so it is sufficient for our illustration to let  $\lambda$  itself adjust, rather than adjusting the number of firms which in turn adjusts  $\lambda$ .

addition is that  $\lambda^*$  must solve  $\frac{(\delta+\rho)b}{\lambda^*}e^{\frac{S^*\rho}{2}-\frac{2\lambda^*}{\rho}}\left(1-e^{-\frac{S^*\rho}{2}}\right) = k$ . If we choose  $k$  so as to maintain the same  $\lambda^*$  as in our calibration, the equilibrium outcome is identical; with substitution of the zero-profit condition, however, we find that  $w_{\text{mean}} = p - k$ .

In the baseline model, an increase in  $b$  allowed firms to offer lower wages as more workers approached expiration of benefits; but this results in higher profits which would attract more firms into the market. With endogenous firm effort, this induces a higher  $\lambda^*$  and a lower  $S^*$ . The net effect causes no change in average wages, though wage dispersion will decrease as the lowest wage rises and the highest falls. Unemployment still rises, as does the average duration of unemployment.

An increase in  $T$  reduces profits in the baseline model; here, this reduces firm effort so  $\lambda^*$  falls and  $S^*$  rises. Average wages are unchanged, but the lowest wage falls while the highest wage rises. The baseline model also led to higher wage dispersion, but there lowest wage was unchanged and the rest of the distribution was stretched upward; here we get a mean-preserving spread. The unemployment rate and expected duration of unemployment both rise.

## 7 Conclusion

Our paper contributes to two strands of literature. First, we show how a finite limit on the duration of unemployment benefits can generate equilibrium wage dispersion among homogeneous workers and firms. The model is solved by translating equilibrium conditions into a differential equation governing reservation wages, a technique that could prove useful in other dynamic programming problems. This provides a tractable environment where we can isolate the incentive effects of unemployment benefits without resorting to on-the-job search or heterogeneity among agents. We provide a thorough characterization of equilibrium, as well as key comparative statics.

Second, our equilibrium framework, which is calibrated to replicate the key labor market statistics in the US, enables us to quantitatively evaluate the effects of extended benefits on the unemployment rate, distribution of wages, unemployment duration, and welfare in the economy. We find that a 10% increase in the size of benefits will lead to a minor (0.4%) decline in wages and major (25.7%) decrease in the hazard rate of leaving unemployment. On the other hand, a 50% extension in potential benefit duration results in a 0.04% increase in wages and a 44% drop in the

hazard rate. These changes in hazard rate have a larger magnitude than most empirical estimates; even so, our model qualitatively replicates the observed pattern of changes quite well. In particular, we explain why hazard rates over the unemployment spell react to policy changes as documented by empirical research, and explain the jump in the hazard rate from unemployment just before benefit expiration. Despite its success in generating dispersed wages in equilibrium, our model fails to account for the extent of dispersion documented by Hornstein, *et al* (2011).

One potential adaptation that may improve the fit of the wage distribution is to allow endogenous search effort by workers, where unemployed workers may engage in costly effort to increase their offer arrival rate. This feature was included in an on-the-job search model in Christensen, *et al* (2005), and dramatically improved the ability of the model to fit observed wage data. Unfortunately, their model does not yield a closed form solution; certainly our environment would be even less analytically tractable. Even so, it may be possible to find numerical results via value function iteration. We leave this investigation for future work.

We have also omitted the ability to self-insure against unemployment spells through precautionary savings, since all workers live hand-to-mouth in our environment. Of course, with risk neutral preferences, such consumption smoothing is unimportant, but with CRRA preferences, Hansen and Imrohoroglu (1992) showed that the ability to save and borrow can reduce the moral hazard created by UI. Similarly, in Shimer and Werning (2008), savings make it optimal to maintain constant UI benefit levels indefinitely, rather than reduce them over the unemployment spell. Both of these studies involve exogenous wages.

The introduction of assets could have even further consequences, as it is possible for savings decisions alone to generate wage dispersion (even with unlimited unemployment benefits). Since employment spells are of random duration, workers who are otherwise identical will enter unemployment with differing asset levels. Bauer (2009) demonstrates that this can produce differing reservation wages under an exogenous distribution of wage offers if workers face a binding credit constraint. We believe this can sustain endogenous wage dispersion in a wage posting environment as well.



# A Proofs

## A.1 Lemma 1

*Proof.* Suppose that the lowest wage offered in the support was  $w = R(z)$  for some  $T > z > 0$ . Note that the reservation wages  $R(s)$  are still well defined for  $s < z$ , even though they are strictly lower than any offered wage. Consider a firm that has encountered a worker and is choosing a wage to offer; its expected profits are  $\Pi(s) \equiv (p - R(s))H(s)$ . We proceed by showing that a firm could earn strictly higher profits by offering a  $R(s)$  just below  $R(z)$ . In other words, we will show that  $\Pi'(z) < 0$ .

Note that the Bellman equations require that  $V_u(s)$  and  $V'_u(s)$  are in  $C^1$ , and since  $V_u(s) = \frac{R(s) + \delta V_u(T)}{\delta + \rho}$ , then  $R(s)$  and  $R'(s)$  are also in  $C^1$ . Inspection of Eq. 7 indicates that  $h'(s)$  is not continuous wherever there are atoms. Even so,  $h(s)$  and  $H(s)$  must be continuous and differentiable as they are just the result of integrating  $h'(s)$ . Thus,  $\Pi(s)$  and  $\Pi'(s)$  are continuous.

If  $\Pi'(z) > 0$ , then targeting an  $s$  slightly higher than  $z$  will earn greater expected profits. Even if  $s$  were in the support, this violates equilibrium condition 2. Note that this does not apply if  $z = T$ .

Suppose instead that  $\Pi'(z) = 0$ . From Eq. 7 and 8, we have  $h'(s) = \lambda h(s)$  and  $h(0) = \lambda H(0)$ , respectively. This differential equation can be solved for  $s \in [0, z)$  as:

$$H(s) = H(0)e^{s\lambda}. \quad (29)$$

Similarly, the Bellman Eq. 2 is  $\rho V_u(0) = x + \lambda(V_{avg} - V_u(0))$ , where  $V_{avg}$  is the average utility from accepted wages:

$$V_{avg} = \int_{R(z)}^{\infty} V_e(w)dF(w).$$

Meanwhile, the Bellman Eq. 3 becomes  $\rho V_u(s) = b + x - V'_u(s) + \lambda(V_{avg} - V_u(s))$ . Note that  $V_{avg}$  recognizes that all offered wages are accepted by workers with  $s < z$  time until expiration. We can solve this differential equation together with its boundary condition at  $s = 0$  above to reach the following solution:

$$V_u(s) = \frac{b(1 - e^{-s(\lambda + \rho)}) + x + \lambda V_{avg}}{\lambda + \rho}.$$

Reservation wages are defined by  $V_u(s) = \frac{R(s) + \delta V_u(T)}{\delta + \rho}$ , so:

$$R(s) = \left( \frac{\delta + \rho}{\lambda + \rho} \right) (b(1 - e^{-s(\lambda + \rho)}) + x + \lambda V_{avg}) - \delta V_u(T). \quad (30)$$

We can now substitute Eqs. 29 and 30 into expected profits, take the derivative w.r.t.  $s$  evaluated at  $s = z$ , and set it equal to zero. This is equivalent to requiring:

$$p + \delta V_u(T) = \frac{\delta + \rho}{\lambda + \rho} \left( b + x + \lambda V_{avg} + \frac{\rho}{\lambda} b e^{-z(\lambda + \rho)} \right).$$

However, if we take the second derivative of  $\Pi(s)$  evaluated at  $s = z$ , we get:

$$\Pi''(z) = \lambda^2 e^{z\lambda} \left( p + \delta V_u(T) - \frac{\delta + \rho}{\lambda + \rho} \left( b + x + \lambda V_{avg} - \frac{\rho^2}{\lambda^2} b e^{-z(\lambda + \rho)} \right) \right) H(0).$$

By substituting for  $p + \delta V_u(T)$  using the  $\Pi'(z) = 0$  condition, we see that  $\Pi''(z) = b\rho(\delta + \rho)b e^{-z\lambda} H(0) > 0$ . Thus, if  $\Pi'(z) = 0$ , this is in fact a minimum rather than a maximum, and expected profits can be increased by offering  $R(s)$  with  $s < z$ , which contradicts equilibrium condition 2.

Thus  $\Pi'(z) < 0$ , so offering a wage just below  $R(z)$  will strictly increase profits. This also contradicts equilibrium condition 2. Recall that it is never profitable to offer more than  $R(T)$  (since all workers will accept less) or less than  $R(0)$  (since all unemployed workers will reject it); thus, the lowest wage in the support must either be  $R(0)$  or  $R(T)$ .  $\square$

## A.2 Constructing the differential equation

In solving for equilibrium, we broadly follow the approach used by Burdett and Mortensen (1998): (1) use the Bellman equations to solve for reservation wages as a function of the wage distribution, (2) use the steady state equations and equal profit condition to back out the wage distribution, and (3) combine these to obtain the final solution. Our key innovation is that the equilibrium conditions can be translated into a differential equation; without this, the problem would remain intractable. This also produces our four boundary conditions, which do not have close analogs in Burdett and Mortensen (1998). Here, we demonstrate this process of translation.

First, we turn to the Bellman equations. The combination of Eqs. 1 and 4

imply that  $V_u(s) = \frac{R(s) + \delta V_u(T)}{\delta + \rho}$ . Taking derivatives, we obtain  $V'_u(s) = \frac{R'(s)}{\delta + \rho}$  and  $V''_u(s) = \frac{R''(s)}{\delta + \rho}$ .

Now consider the Bellman Eq. 3 in the range where unemployed workers accept some job offers,  $s \in (0, S^*)$ . We substitute for  $V_e(w)$  using Eq. 1 and then take the derivative w.r.t.  $s$  to obtain the differential equation:<sup>22</sup>

$$(\rho + \lambda(1 - F(R(s)))) V'_u(s) = -V''_u(s) \implies (\rho + \lambda(1 - F(R(s)))) R'(s) = -R''(s). \quad (31)$$

Next, we establish a boundary condition at  $s = 0$ . Consider Eq. 3 as  $s \rightarrow 0$ :

$$\begin{aligned} \rho V_u(0) = & b + x - V'_u(0) \\ & + \lambda \left( V_e(w_h) \mu_F(w_h) + \int_{R(0)}^{w_h} V_e(w) f(w) dw - (1 - \mu_F(w_\ell)) V_u(0) \right), \end{aligned}$$

where  $w_\ell = R(0)$  and  $w_h = R(S^*)$ .

At the same time, Eq. 2 defines the Bellman equation at  $s = 0$ . Since benefits are lost and all wages are accepted, we have:

$$\rho V_u(0) = x + \lambda \left( V_e(w_h) \mu_F(w_h) + \int_{w_\ell}^{w_h} V_e(w) f(w) dw + V_e(w_\ell) \mu_F(w_\ell) - V_u(0) \right).$$

However, bearing in mind that  $V_e(w_\ell) = V_u(0)$  and  $w_\ell = R(0)$ , the equation above simplifies to:

$$\rho V_u(0) = x + \lambda \left( V_e(w_h) \mu_F(w_h) + \int_{R(0)}^{w_h} V_e(w) f(w) dw - (1 - \mu_F(w_\ell)) V_u(0) \right).$$

Thus,  $V_u$  is continuous at  $s = 0$  if and only if  $V'_u(0) = b$ , which is equivalent to:

$$R'(0) = b(\delta + \rho). \quad (32)$$

The continuity of  $V_u$  also imposes a boundary condition at  $s = S^*$ . In the range where no wages are accepted,  $s \in (S^*, T]$ , Eq. 3 is a first-order differential equation,  $\rho V_u(s) = b + x - V'_u(s)$ , with boundary condition  $V_u(S^*) = \frac{R(S^*) + \delta V_u(T)}{\delta + \rho}$  from Eq. 1.

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<sup>22</sup>Theorem 1 of van den Berg (1990) develops a similar differential equation for the reservation wage function. Indeed, our Eq. 31 is the first derivative of that differential equation, which eliminates the integral and thus facilitates solving for an endogenous distribution  $F(w)$ .

This has the solution:

$$V_u(s) = \frac{b+x}{\rho} - \frac{(b+x-R(S^*))e^{\rho(S^*-s)}}{\delta(1-e^{\rho(S^*-T)})+\rho}. \quad (33)$$

On the other hand, when Eq. 3 is evaluated at  $s = S^*$ , it becomes:

$$\rho V_u(S^*) = b+x - V'_u(S^*) + \lambda(V_e(w_h) - V_u(S^*))\mu_F(w_h).$$

However,  $w_h = R(S^*)$  and  $V_e(R(S^*)) = V_u(S^*)$ , so the last term cancels, yielding  $\rho V_u(S^*) = b+x - V'_u(S^*)$ . By substituting in  $V'_u(S^*) = \frac{R'(S^*)}{\delta+\rho}$  and  $V_u(S^*)$  from Eq. 33, we obtain:

$$R'(S^*) = \frac{\rho(\delta+\rho)(b+x-R(S^*))}{\delta(1-e^{\rho(S^*-T)})+\rho}. \quad (34)$$

We then turn to the equal profit condition and the steady state conditions. First, since unemployed workers with  $s \in (S^*, T]$  reject all wage offers,  $F(R(s)) = 1$  in this range and hence Eq. 7 implies  $h'(s) = 0$ . Thus  $h(s) = h(T) = \frac{\delta(1-u)}{u}$ , from Eq. 6. Moreover, since  $H(T) = 1$ ,  $H(S^*) = 1 - \frac{\delta(1-u)}{u}(T - S^*)$ .

Again, we proceed by characterizing reservation wages for  $s \in [0, S^*]$ . Eq. 10 specifies equal profits for all wages offered in equilibrium. In particular, at  $w_h$ ,  $H(R^{-1}(w_h)) = H(S^*)$ , so:

$$\pi = \frac{\lambda u}{(1-u)\delta}(p-w_h) \left( 1 - \frac{\delta(1-u)}{u}(T-S^*) \right).$$

Since all wages are equally profitable, this must also equal  $\frac{\lambda u}{(1-u)\delta}(p-w)H(R^{-1}(w))$  for any  $w \in [w_\ell, w_h]$ . Therefore,

$$H(R^{-1}(w)) = \frac{(u-(1-u)(T-S^*)\delta)(p-w_h)}{u(p-w)}.$$

By substituting  $w = R(s)$ , this becomes:  $H(s) = \frac{(u-(1-u)(T-S^*)\delta)(p-R(S^*))}{u(p-R(s))}$ , with derivatives:

$$h(s) = \frac{(u-(1-u)(T-S^*)\delta)(p-R(S^*))R'(s)}{u(p-R(s))^2} \quad \text{and}$$

$$h'(s) = \frac{(u-(1-u)(T-S^*)\delta)(p-R(S^*))(2(R'(s))^2 + (p-R(s))R''(s))}{u(p-R(s))^3}.$$

These can then be substituted into the steady state conditions (Eqs. 6, 7, and 8)

to obtain:

$$h(S^*) = \frac{\delta(1-u)}{u} \implies R'(S^*) = \frac{\delta(1-u)}{u - \delta(1-u)(T - S^*)}(p - R(S^*)) \quad (35)$$

$$h'(s) = \lambda(1 - F(R(s)))h(s) \implies F(R(s)) = 1 - \frac{2R'(s)}{\lambda(p - R(s))} - \frac{R''(s)}{\lambda R'(s)} \quad (36)$$

$$h(0) = \lambda H(0) \implies R'(0) = \lambda(p - R(0)). \quad (37)$$

To conclude, we combine the steady state and Bellman equation translations into a single system. Using Eq. 36 to substitute for  $F(R(s))$  in Eq. 31, we obtain the following second-order differential equation for  $R(s)$ :

$$\rho R'(s) + 2R''(s) + \frac{2(R'(s))^2}{p - R(s)} = 0. \quad (38)$$

When solved using Eqs. 32 and 37 as boundary conditions, we obtain  $R(s)$  as depicted in Eq. 13 for  $s \in [0, S^*]$ . Eq. 35 determines the unemployment rate, and Eq. 34 is equivalent to Eq. 11, *i.e.*  $\phi(S^*) = 0$ .

The remaining equilibrium objects follow from this solution. Having obtained  $R(s)$ , one can use Eq. 36 to solve for  $F(R(s))$ . Similarly,  $H(s)$  is specified above as a function of  $R(s)$ ; and  $g(s)$  is computed from Eq. 9. Finally, we must obtain the implicit reservation wage for unemployed workers with remaining time  $s \in (S^*, T]$ . This comes from substituting  $V_u(s) = \frac{R(s) + \delta V_u(T)}{\delta + \rho}$  into our solution to  $V_u(s)$  in Eq. 33. By solving for  $R(s)$  one reaches the formula in Eq. 13.

### A.3 Lemma 2

*Proof.* We proceed by establishing the following four claims:

1. If there exists a solution  $S^* \in [0, T]$  such that  $\phi(S^*) = 0$ , then  $\phi'(S^*) < 0$ , and  $S^*$  is the only such solution.
2. If  $\phi(0) \leq 0$ , then  $\phi(S) < 0$  for all  $S \in (0, T]$ .
3. If  $\phi(T) > 0$ , then  $\phi(S) > 0$  for all  $S \in [0, T]$ .
4. If  $\phi(0) > 0$  and  $\phi(T) \leq 0$ , then there exists a unique  $S^* \in [0, T]$  such that  $\phi(S^*) = 0$ .

The antecedents of claims two through four are mutually exclusive, and  $\phi(T) > 0$  contradicts Assumption 2; thus we reach the lemma's conclusion.

First, consider claim 1. Assume that there is a  $S^* \in [0, T]$  such that  $\phi(S^*) = 0$ . The first derivative of  $\phi$  is:

$$\phi'(S) = \frac{1}{2} e^{-\frac{S\rho}{2}} \left( \frac{2(p-b-x)\lambda}{b} e^{-\frac{2\lambda}{\rho}(-1+e^{-\frac{S\rho}{2}})} - \delta - e^{(S-T)\rho} \delta - \rho \right). \quad (39)$$

If evaluated at  $S^*$ , we may use  $\phi(S^*) = 0$  to substitute for  $e^{-\frac{2\lambda}{\rho}(-1+e^{-\frac{S^*\rho}{2}})}$ , obtaining:

$$\phi'(S^*) = \frac{e^{-S^*\rho} (\delta (1 - e^{(S^*-T)\rho}) + \rho) (\rho e^{\frac{S^*\rho}{2}} - 2\lambda)}{2\rho}. \quad (40)$$

Since we only consider  $S^* \in [0, T]$ ,  $1 - e^{(S^*-T)\rho} \geq 0$  and  $e^{\frac{S^*\rho}{2}} \leq e^{\frac{T\rho}{2}} < \frac{\lambda}{\rho} < \frac{2\lambda}{\rho}$ , where the second-to-last inequality comes from Assumption 1. Thus,  $\phi'(S^*) < 0$  wherever  $\phi(S^*) = 0$ . Since  $\phi$  is continuous, there is at most one  $S^*$  such that  $\phi(S^*) = 0$ .

The second and third claims are simple extensions. If  $\phi(0) < 0$  and  $\phi$  is continuous, there cannot be a  $\hat{S} \in [0, T]$  such that  $\phi(\hat{S}) \geq 0$  unless there exists a  $S^* \in [0, \hat{S}]$  such that  $\phi(S^*) = 0$  and  $\phi'(S^*) > 0$ , which contradicts. Similarly, if  $\phi(T) > 0$ , there cannot be a  $\hat{S} \in [0, T]$  such that  $\phi(\hat{S}) \leq 0$  unless there exists a  $S^* \in [\hat{S}, T]$  such that  $\phi(S^*) = 0$  and  $\phi'(S^*) > 0$ , which contradicts.

The fourth claim follows from the continuity of  $\phi$ . □

## A.4 Proposition 1

*Proof.* We will show that Eqs. 13 through 19 satisfy all of the equilibrium conditions. We start by examining the steady state Eqs. 6 through 9. Of course, as we described in Appendix A.2,  $h(s)$ ,  $g(w)$ , and  $f(w)$  were constructed to satisfy these conditions given the solution to  $R(s)$ ; to be thorough, we verify here that we were successful. First, we need the p.d.f.s of the equilibrium distributions, which are found by taking

the first derivative of Eqs. 14, 16, and 17, respectively:

$$f(w) = \begin{cases} \frac{\rho}{2\lambda(p-w)} & w \in (w_\ell, w_h) \\ 0 & w \in [0, w_\ell] \cup (w_h, \infty) \end{cases} \quad (41)$$

$$h(s) = \begin{cases} \frac{\lambda e^{\frac{2\lambda}{\rho} \left( e^{-\frac{S^*\rho}{2}} - e^{-\frac{s\rho}{2}} \right) + \frac{(S^*-s)\rho}{2}}}{e^{\frac{S^*\rho}{2}} + \lambda(T-S^*)} & s \leq S^* \\ \frac{\lambda}{e^{\frac{S^*\rho}{2}} + \lambda(T-S^*)} & s > S^* \end{cases} \quad (42)$$

$$g(w) = \begin{cases} \frac{\rho(\delta+\rho)b}{2(p-w)^2\lambda^2} e^{\frac{S^*\rho}{2} - \frac{2\lambda}{\rho} \left( 1 - e^{-\frac{S^*\rho}{2}} \right)} & w \in (w_\ell, w_h) \\ 0 & w \in [0, w_\ell] \cup (w_h, \infty) \end{cases} \quad (43)$$

where  $w_\ell = R(0)$  and  $w_h = R(S^*)$ .

Using these formulas and minor algebraic manipulation, one can quickly verify that steady state Eqs. 6 through 8 (governing transitions into and through unemployment) are satisfied. We turn our attention to verifying Eq. 9, which is more involved. Note that, dividing Eq. 43 by Eq. 41 yields:

$$\frac{g(w)}{f(w)} = \frac{(\delta + \rho)b}{(p - w)\lambda} e^{\frac{S^*\rho}{2} - \frac{2\lambda}{\rho} \left( 1 - e^{-\frac{S^*\rho}{2}} \right)} \text{ for } w \in [w_\ell, w_h]. \quad (44)$$

Similarly,

$$\frac{\mu_G(w_\ell)}{\mu_F(w_\ell)} = e^{\frac{S^*\rho}{2} - \frac{2\lambda}{\rho} \left( 1 - e^{-\frac{S^*\rho}{2}} \right)} \text{ and } \frac{\mu_G(w_h)}{\mu_F(w_h)} = e^{\frac{S^*\rho}{2}}.$$

We also need to compute  $H(R^{-1}(w))$  from Eq. 16 and the inverse of Eq. 13, which after algebraic manipulation, becomes:

$$H(R^{-1}(w)) = \frac{b(\delta + \rho)}{(p - w)\lambda \left( e^{\frac{S^*\rho}{2}} + (T - S^*)\lambda \right)} e^{\frac{S^*\rho}{2} - \frac{2\lambda}{\rho} \left( 1 - e^{-\frac{S^*\rho}{2}} \right)}.$$

Substituting  $\frac{g(w)}{f(w)}$  and  $H(R^{-1}(w))$  into Eq. 9, we get:

$$\delta \frac{(\delta + \rho)b}{(p - w)\lambda} e^{\frac{S^*\rho}{2} - \frac{2\lambda}{\rho} \left( 1 - e^{-\frac{S^*\rho}{2}} \right)} = \frac{\delta \left( e^{\frac{S^*\rho}{2}} + (T - S^*)\lambda \right) b(\delta + \rho)}{(p - w)\lambda \left( e^{\frac{S^*\rho}{2}} + (T - S^*)\lambda \right)} e^{\frac{S^*\rho}{2} - \frac{2\lambda}{\rho} \left( 1 - e^{-\frac{S^*\rho}{2}} \right)},$$

which holds for all  $w$ . We also verify that the same equation holds for both atoms in the distribution. At  $w_h$ , we have:

$$\delta e^{\frac{S^*\rho}{2}} = \lambda \frac{\delta \left( e^{\frac{S^*\rho}{2}} + (T - S^*)\lambda \right)}{\lambda} \frac{e^{\frac{S^*\rho}{2}}}{e^{\frac{S^*\rho}{2}} + (T - S^*)\lambda},$$

and at  $w_\ell$ ,

$$\delta e^{\frac{S^*\rho}{2} - \frac{2\lambda}{\rho} \left( 1 - e^{-\frac{S^*\rho}{2}} \right)} = \lambda \frac{\delta \left( e^{\frac{S^*\rho}{2}} + (T - S^*)\lambda \right)}{\lambda} \frac{e^{\frac{S^*\rho}{2} - \frac{2\lambda}{\rho} \left( 1 - e^{-\frac{S^*\rho}{2}} \right)}}{e^{\frac{S^*\rho}{2}} + (T - S^*)\lambda},$$

which both hold.

Next, we show that the equal-profit condition holds for all wages in the support of  $F$ , and that wages outside the support are no more profitable. We note that while the former was used in Appendix A.2 to construct our solution, the latter was not and thus requires the proof which follows. Steady state profits are  $(p - w) \frac{g(w)}{f(w)}$ , into which we may substitute Eq. 44 above to yield the proposed equilibrium profit in Eq. 18, which is constant for all  $w \in [w_\ell, w_h]$ .

Moreover, all wages outside the support will result in no greater profit. Consider if one infinitesimal firm deviated, offering a wage outside the support but not changing the distribution of wage offers. Offering a wage below  $w_\ell$  would result in zero profit, since all unemployed workers have a reservation wage at or above  $w_\ell$ , given  $F$ .

On the other hand, if the firm offers a wage  $w = R(s) > R(S^*) = w_h$ , they would be able to successfully attract additional workers (with between  $S^*$  and  $s$  time until expiration), but this also reduces realized profit on those who do accept. We will show that the net of these two effects will result in lower profit.

Recall that unemployed workers with  $s > S^*$  time until benefit expiration require  $R(s) = p - \frac{(p-b-x)(\delta+\rho)}{\delta\lambda e^{(S^*-T)\rho} + (\delta+\rho) \left( \rho e^{\frac{S^*\rho}{2}} - \lambda \right)}$  as a reservation wage. The fraction of unemployed workers willing to accept such an offer is  $H(s) = \frac{1 + \lambda(s - S^*)e^{-\frac{S^*\rho}{2}}}{1 + \lambda(T - S^*)e^{-\frac{S^*\rho}{2}}}$ . Thus, the expected profit from offering a wage targeted for  $s$  is  $(p - R(s))H(s)$ , which becomes:



$$\Pi(s) \equiv \frac{\left(e^{\frac{S^*\rho}{2}} + \lambda(s - S^*)\right) (p - b - x)(\delta + \rho) \left(\rho e^{\frac{S^*\rho}{2}} - \lambda(1 - e^{(S^*-s)\rho})\right)}{\left(e^{\frac{S^*\rho}{2}} + \lambda(T - S^*)\right) \left(\delta \lambda e^{(S^*-T)\rho} + (\delta + \rho) \left(\rho e^{\frac{S^*\rho}{2}} - \lambda\right)\right)}.$$

At  $s = S^*$ ,  $\Pi(S^*)$  provides the same expected profit that is experienced by offering a wage in the support of  $F$ . Note that this expected profit of offering a particular wage (as discussed in Section 2.3) differs from average steady state profit only by the constant  $\frac{\lambda u}{\delta(1-u)}$ . Its first derivative is:

$$\Pi'(s) = -\frac{\lambda(p - b - x)(\delta + \rho) \left(\lambda - \rho e^{\frac{S^*\rho}{2}} (1 - e^{(S^*-s)\rho}) - \lambda e^{(S^*-s)\rho} (1 + \rho(S^* - s))\right)}{\left(e^{\frac{S^*\rho}{2}} + \lambda(T - S^*)\right) \left(\delta \lambda e^{(S^*-T)\rho} + (\delta + \rho) \left(\rho e^{\frac{S^*\rho}{2}} - \lambda\right)\right)}.$$

If  $\phi(S^*) = 0$  (*i.e.* this is a dispersed equilibrium), then the first derivative of  $\Pi$  evaluated at  $s = S^*$  is equal to zero. The second derivative of  $\Pi$  evaluated at  $s = S^*$  yields:

$$\Pi''(S^*) = -\frac{\rho \lambda (p - b - x)(\delta + \rho) \left(2\lambda - \rho e^{\frac{S^*\rho}{2}}\right)}{\left(e^{\frac{S^*\rho}{2}} + \lambda(T - S^*)\right) \left(\delta \lambda e^{(S^*-T)\rho} + (\delta + \rho) \left(\rho e^{\frac{S^*\rho}{2}} - \lambda\right)\right)} < 0.$$

Thus, as a firm attempts to target  $s$  near but greater than  $S^*$ , his expected profits will fall. We must also verify that  $\Pi'(s) < 0$  for all  $s > S^*$ . Substituting for  $p - x$  using  $\phi(S^*) = 0$ , this is equivalent to saying:

$$(e^{s\rho} - e^{S^*\rho}) \left(\lambda - \rho e^{\frac{S^*\rho}{2}}\right) + \lambda \rho (s - S^*) e^{S^*\rho} > 0.$$

The first term is positive because of Assumption 1, and the last term is clearly positive as well.

On the other hand, if  $\phi(S^*) < 0$  (so  $S^* = 0$ ), then

$$\Pi'(0) = \frac{\lambda}{1 + \lambda T} \left(p - x - b \left(\frac{\delta + \rho}{\lambda} - \frac{\delta}{\rho} (1 - e^{-T\rho})\right)\right).$$

In fact, comparison to Eq. 12 reveals that  $\Pi'(0) < 0$  if and only if  $\phi(S^*) < 0$ , which is the necessary condition for the existence of a degenerate equilibrium. Verifying that

$\Pi'(s) < 0$  for all  $s > S^*$  proceeds just as above.

The only remaining task is to verify that  $R(s)$  maximizes the utility of a worker with  $s$  time until expiration. First, we check that our Bellman Eqs. 2 and 3 are satisfied given  $R(s)$ . One could do this by substituting for  $R(s)$  and  $F(w)$  into the literal definitions and evaluating the integrals, though it results in large amounts of algebraic manipulation. Yet in Appendix A.2, we showed that those equations are equivalent to Eq. 31 (for  $s \in (0, S^*]$ ), Eq. 32 (for  $s = 0$ ), and Eq. 33 (for  $s \in (S^*, T]$ ). Thus, a more elegant approach is to demonstrate that we satisfy this system of differential equations.

First, consider when  $s \in (0, S^*]$ . In this range, the first and second derivatives of  $R(s)$  are:

$$R'(s) = (\delta + \rho)be^{-\frac{s\rho}{2} - \frac{2\lambda}{\rho}}(1 - e^{-\frac{s\rho}{2}}).$$

and

$$R''(s) = -(\delta + \rho)be^{-\frac{s\rho}{2} - \frac{2\lambda}{\rho}}(1 - e^{-\frac{s\rho}{2}}) \left( \frac{\rho}{2} + \lambda e^{-\frac{s\rho}{2}} \right).$$

If we substitute for  $F(R(s))$  using Eq. 15, then Eq. 31 requires that  $(\frac{\rho}{2} + \lambda e^{-\frac{s\rho}{2}}) R'(s) = -R''(s)$ , which holds.

Next, consider when  $s = 0$ . Here, Eq. 32 requires that  $R'(0) = b(\delta + \rho)$ , which also holds.

Finally, for  $s \in (S^*, T]$ , the Bellman Eq. 3 is equivalent to  $\rho V_u(s) = b + x - V_u'(s)$ . We can still use  $V_u(s) = \frac{R(s) + \delta V_u(T)}{\delta + \rho}$  and  $V_u'(s) = \frac{R'(s) + \delta V_u'(T)}{\delta + \rho}$  to rewrite this in terms of the implicit reservation wage. Also, since  $V_u(T) = \frac{R(T) + \delta V_u(T)}{\delta + \rho}$ , then  $V_u(T) = \frac{R(T)}{\rho}$ . Thus, the Bellman equation is translated to:

$$\frac{R(s) + \frac{\delta}{\rho}R(T)}{\delta + \rho} = \frac{b + x}{\rho} - \frac{R'(s)}{\rho(\delta + \rho)}.$$

In this range of  $s$ ,  $R'(s) = \frac{e^{(S^*-s)\rho}(p-b-x)\lambda\rho(\delta+\rho)}{\delta\lambda e^{(S^*-T)\rho} + (\delta+\rho)\left(\rho e^{\frac{S^*\rho}{2}} - \lambda\right)}$ . Substituting  $R'(s)$ ,  $R(s)$ , and  $R(T)$ , this becomes:

$$\frac{p}{\rho} - \frac{p-b-x}{\rho} \left( \frac{\delta \lambda e^{(S^*-T)\rho} + (\delta + \rho) \rho e^{\frac{S^*\rho}{2}} - (\delta + \rho) \lambda + \lambda \rho e^{(S^*-s)\rho}}{\delta \lambda e^{(S^*-T)\rho} + (\delta + \rho) \left( \rho e^{\frac{S^*\rho}{2}} - \lambda \right)} \right) = \frac{b+x}{\rho} - \frac{e^{(S^*-s)\rho} (p-b-x) \lambda}{\delta \lambda e^{(S^*-T)\rho} + (\delta + \rho) \left( \rho e^{\frac{S^*\rho}{2}} - \lambda \right)}.$$

Note that, in the parenthetical term on the left-hand side of the equation, the first three terms in the numerator are equivalent to the denominator. After this is simplified, both sides of the equation are identical.

Thus, the Bellman equations are satisfied by this construction. Note also that the value functions are continuous at  $s = S^*$  (given Eq. 11) and  $s = 0$ . Finally,  $R(s)$  is the optimal reservation wage at each  $s$ , because workers are exactly indifferent about accepting that wage:  $V_e(R(s)) = V_u(s)$ . If a higher reservation wage  $w$  were chosen, the worker would pass up wages between  $R(s)$  and  $w$ , even though these are strictly better than being unemployed (since  $V_e(w)$  is strictly increasing in  $w$ ). By the same token, a lower reservation wage would have workers accept wages between  $w$  and  $R(s)$  that provide a strictly lower flow of utility than remaining unemployed.

□

## A.5 Lemma 3

*Proof.* In the proof of Lemma 2 (Section A.3), we established that if a dispersed equilibrium exists, then  $\frac{\partial \phi}{\partial S} < 0$  when evaluated at  $S^*$ . In addition, the partial derivatives with respect to  $b$  and  $T$  are:

$$\frac{\partial \phi}{\partial b} = -\frac{p-x}{b^2} e^{\frac{2\lambda}{\rho} \left( 1 - e^{-\frac{S^*\rho}{2}} \right)} < 0 \quad \text{and} \quad \frac{\partial \phi}{\partial T} = \delta e^{\frac{1}{2}(S^*-2T)\rho} > 0.$$

Thus, by implicit differentiation,  $\frac{\partial S^*}{\partial b} = -\frac{\partial \phi / \partial b}{\partial \phi / \partial S} < 0$  and  $\frac{\partial S^*}{\partial T} = -\frac{\partial \phi / \partial T}{\partial \phi / \partial S} > 0$ . □

## A.6 Equilibria when Assumption 2 is violated

When benefits are not sufficiently generous (*i.e.* when  $\phi(T) > 0$ ), neither equilibrium presented in Section 3 exists. Even so, it is still possible to find an equilibrium solution, which we refer to as *early equilibria* because workers begin accepting at

least some job offers immediately on entering an unemployment spell (*i.e.*  $R(T)$  is in the support  $F$ ). This is contrasted with the *late equilibria* depicted in Section 3, where workers wait until later in their spell before any of the offers are acceptable.

Here, we provide a brief roadmap of how to solve for early equilibria. First, it is possible that  $R(T)$  is the only wage offered in a degenerate equilibrium. Since this is the only wage offered ( $F(R(s)) = 1$ ), it is quite simple to solve the Bellman Eqs. 2 and 3 and the steady state equations. The only issue is to verify that it is not profitable for firms to deviate, offering wages  $R(s)$  for  $s < T$ . This turns out to be true only if an additional condition is met, which requires benefits to be quite low.

It is also possible to have early dispersed equilibria, which bear resemblance to the late dispersed equilibria, with a few significant complications. Here, the support of  $F$  will span from  $R(Z^*)$  to  $R(T)$ , where  $Z^* \in [0, T]$ . However, as noted in Lemma 1,  $R(0)$  must be part of the support as well, with an atom of size  $1 - F(R(0)) = a^*$ . Thus, firms are targeting workers early in their unemployment spell and workers whose benefits have expired, but may ignore those whose time until expiration is between 0 and  $Z^*$ . The distribution  $F$  also will include atoms at  $R(Z^*)$  and  $R(T)$ .

The process of solving for  $R$  and  $F$  is very similar to that described in Appendix A.2. For  $s \in (Z^*, T)$ , the differential equation remains the same, and four boundary conditions are imposed to ensure that  $V_u(s)$  and  $H(s)$  are continuous at  $s = Z^*$  and  $s = T$ . As with  $S^*$  in late equilibria,  $Z^*$  must be implicitly solved from these boundary conditions. The early equilibrium also requires two additional boundary conditions to ensure that  $V_u(s)$  and  $H(s)$  are continuous at  $s = 0$ . These are used to find  $R(0)$  and  $a^*$ ; again, the latter can only be solved for implicitly.

The equilibrium outcome is thus presented in terms of  $Z^*$  and  $a^*$ . This system of implicit solutions is much less tractable than the single equation for  $S^*$  in the late equilibrium. We have verified, however, that as  $Z^*$  approaches 0, so does  $a^*$ . Thus, on this boundary, the early equilibrium coincides with the full support late dispersed equilibrium where  $S^* = T$ .

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