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Polarization and pandering in common-interest elections

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ABSTRACT

Adding candidates to a one-dimensional common-interest voting model, this paper shows that catering to centrist voters can lower social welfare. The electoral benefit of doing so is weak, so candidates polarize substantially in equilibrium, resolving a long-standing empirical puzzle.

1. Introduction

Standard spatial election models assume that voters intrinsically prefer different policies within an interval. To appeal to as many voters as possible, political candidates cater to the median voter, whether they are motivated to win, or to control policy, which requires winning first (Hotelling, 1929; Downs, 1957; Calvert, 1985). This paper instead explores a common interest model, where voters function as armchair social planners, seeking to promote overall welfare rather than their idiosyncratic interests, but hold different opinions as to which policy is best.[2] Democracy then plays a truth-seeking role, as in Condorcet's (1785) classic "jury" theorem, selecting policies endorsed by the largest number of thinkers, rather than the directly utilitarian role of privileging the largest group of interests. McMurray (2017a) adapts Condorcet's binary voting model to a spatial environment; by adding political candidates, this paper endogenizes the policy menu voters face.

A well known result in common interest elections is that a vote only affects utility if it is pivotal, making or breaking a tie, and a voter who restricts attention to this event behaves as if conditioning on other voters' private information. The

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[2] As I explain in McMurray (2013), this can occur as large elections amplify even slight voter altruism.

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first result below is that a candidate faces a pivotal calculus, too, because her platform only matters if she wins. Restricting attention to this event, candidates adjust their behavior as if inferring private information from voters.

If candidates are welfare motivated, this pivotal calculus is polarizing: even if they have no private information of their own, so that both are inclined to adopt the same centrist policy, one takes a position left of center, reasoning that if she wins it will be because voters received private information that the optimal policy is left of center, while the other reasons oppositely and take a position on the right. Basing their behavior on large numbers of (anticipated) voter signals, candidates can be quite extreme—potentially more extreme than any voter.

When candidates polarize, there are voters who think the best policy lies between the candidates' platforms. By moving toward her opponent, a candidate can win these voters over and increase her vote share. This leads to a *median opinion theorem*: in equilibrium, candidates who care more about winning polarize less; if they care enough about winning, candidates adopt identical, centrist platforms. This is the same behavior predicted in private interest settings, where a median voter theorem reflects utilitarian compromise between competing voter interests. Here, however, it can produce policies that everyone knows are undesirable, thus amounting to a spatial form of *pandering* to voters' premature opinions, reminiscent of the binary models of Harrington (1993), Canes-Wrone et al. (2001), Heidhues and Lagerlöf (2003), and Maskin and Tirole (2004) but with geometry that those binary models do not exhibit.

Strategic voting generates an underdog effect that favors extreme candidates, so in large elections, majority opinion settles on whichever policy platform is truly superior. A good platform will therefore prevail, even if it is extreme, and a poor platform will lose, even if it is moderate. In other words, centrist platforms give only a limited electoral advantage. The main result of the paper is that, because of this, candidates still polarize substantially in equilibrium *even* when they want very badly to win.

These results are useful because, empirically, Hall (2015) presents causal evidence that elections penalize extremists, but numerous studies find that extremists still frequently win (e.g. Ansolabehere et al., 2001; Canes-Wrone et al., 2002; and Cohen et al., 2016), and that, despite any electoral disadvantage, elected officials in the U.S. House, Senate, presidency, and state legislatures behave similarly to the most extremely liberal and conservative voters in the electorate (Poole and Rosenthal, 1984; Alvarez and Nagler, 1995; McCarty and Poole, 1995; Ansolabehere et al., 2001; Jessee, 2009, 2010, 2016; Bafumi and Herron, 2010; Shor, 2011; and Fowler and Hall, 2016). This matches voter perception: across twelve U.S. presidential elections, 89% of Americans rated both major candidates as weakly more extreme than they rated themselves on a seven-point scale from liberal to conservative, while only 12% rated both candidates as weakly more moderate.[3]

In contrast with the empirical evidence above, the robust prediction of private interest models is that candidates should cater to the center. Pundits often ascribe polarization to candidates' need to satisfy extremist donors, activists, or primary election voters, for example, but in a standard framework, these elites should encourage moderation, recognizing that extremism will merely sacrifice victory and policy control to the opposing side.[4] From a private interest perspective, extreme policy outcomes are also bad for welfare, so empirical polarization is not only puzzling but troubling, reflecting some inexplicable failure of democracy. A possible policy remedy might be to raise candidate salaries, to increase office motivation and foster compromise.

The analysis below offers new perspective on these issues. Candidates can polarize in equilibrium even when they are highly motivated to win, so increasing salaries may have little effect. Sufficiently office motivated candidates do adopt centrist platforms, but this can reduce welfare. The new geometry of pandering helps explain why many people criticize centrist candidates for compromising their principles, but laud extremists as confident visionaries.[5] Section 5 conjectures that extending the common interest setting might generate excessive polarization, but below, equilibrium polarization enhances welfare, giving voters the best possible binary policy menu.[6]

2. Related literature

Martinelli (2001), Heidhues and Lagerlöf (2003), Laslier and Van der Straeten (2004), Gratton (2014), Kartik et al. (2015), and Prato and Wolton (2018) all study candidate positioning in at least partly common interest environments but focus on whether candidates reveal their private information to voters. Candidates in Razin (2003) and McMurray (2017b) respond to voter information, but *after* voting takes place, making inference from realized vote totals. This paper seems to be the first to focus on voter information with candidates moving first.

Though not explicitly common interest, probabilistic voting literature already foreshadows the link between common interest and polarization. Wittman (1983) points out that candidates polarize if they are policy motivated and uncertain

3 American National Election Studies (ANES), 1972–2016.

4 Private interest literature offers numerous theories of polarization—too many to review here—but convergence is sufficiently persistent that Roemer (2004) refers to the "tyranny of the median voter theorem.".

5 In the U.S., for example, centrists are sometimes disparaged as DINO or RINO (Democrats- or Republicans-in-name-only). The 2000 Green party presidential candidate, Ralph Nader, criticized the more centrist Republicans and Democrats as "look alike parties", "Tweedledum and Tweedledee" (http://www.cbsnews.com/news/nader-assails-major-parties).

6 This also makes sense of Tocqueville's (1835, p. 175) praise for political parties that "cling to principles rather than to their consequences" and an American Political Science Association (1950) manifesto advocating to "keep parties apart" and remember that "putting a particular candidate into office is not an end in itself": in the context of this paper, these can be understood as calls for welfare motivation over office motivation.

of the location of the median voter, but Calvert (1985) clarifies that slight uncertainty generates only slight polarization. If voter ideal points $\hat{x}_i$ are i.i.d. from a known distribution, for example, candidates converge asymptotically as n grows large. Permutations of the vector of ideal points should be equally likely, so if $\hat{x}_i$ are correlated, their distribution is exchangeable, and by De Finetti's (1980) theorem, can be decomposed as $\hat{x}_i = z + \varepsilon_i$, where ε_i are i.i.d. conditional on a latent variable z. Indeed, this is a common specification for probabilistic voting models. Polarization is substantial only if z has high variance (relative to ε_i).[7] Since z also shifts all voters' ideal points in tandem, however, it amounts to a variable of common interest; if its variance is high (relative to ε_i) then common interest is dominant in voters' motivations, just as below.[8]

Polarization is even more pronounced below than in probabilistic voting models, for several reasons. In shifting voters' ideal points, the z in probabilistic voting models is effectively common knowledge, and idiosyncratic ε_i makes many voters prefer policies more extreme than the most extreme realizations of z. Below, no voter prefers a policy more extreme than z, and uncertainty pushes voter expectations toward their (centrist) prior, making candidates extreme by comparison. Candidates are also objectively more extreme, both because inference from so many voters gives them confidence, and because the underdog effect of strategic voting favors extremists. A welfare motivated candidate also wants the best policy even if it differs from her own platform, while a policy motivated candidate detests her opponent's platform, and so more willingly compromises to win, for any level of office motivation.[9]

3. The model

There are N voters and two political candidates, A and B. As in Myerson (1998), N is drawn from a Poisson distribution with mean n. Together, these voters and candidates implement a policy x from a (possibly infinite) interval X. One policy $z \in X$ maximizes welfare, and voters and candidates all prefer policies as close as possible to z.[10] However, the location of z is unknown: at the beginning of the game, z is drawn from a known distribution F with density f that is *log-concave*.[11] For ease of exposition, $f(-z) = f(z)$ so that f is also *symmetric* around the origin.

In a first stage, the two candidates simultaneously commit to policy platforms $x_A, x_B \in X$. Observing these, voters each vote simultaneously for one of the two candidates. The candidate $w \in \{A, B\}$ who receives more votes (breaking a tie if necessary by a fair coin toss) wins the election and implements her platform. If x is implemented when z was optimal, each voter receives utility

$$u(x, z) = -(x - z)^2 \tag{1}$$

that declines quadratically with the distance between x and z.[12] Candidate j receives the same, but with an additional benefit β if she wins office.[13]

$$u_j(x, z) = u(x, z) + 1_{w=j}\beta \tag{2}$$

A candidate is *welfare motivated* if $\beta = 0$ but becomes more *office motivated* as β grows.

Before voting, each voter observes a private signal s_i, drawn from the interval S according to a known distribution $G(s|z)$ with density $g(s|z)$. Conditional on z, these private signals are jointly independent. The family of conditional distributions $G(s|z)$ satisfy the *monotone likelihood ratio property (MLRP)*, meaning that $\frac{g(s'|z)}{g(s|z)}$ increases in z, for all $s' > s$. For ease of exposition, g is also *symmetric*, meaning that $g(-s|-z) = g(s|z)$ for any s and z. For any z, $g(s|z)$ also has full support on S. Though voters share a common interest, a spectrum of private signals translates by Bayes' rule into a spectrum of private opinions regarding which policy is optimal.[14]

It would be natural to assume that candidates observe private signals of z, just as voters do. In equilibrium, candidates should also infer anything they can from voters. To make this inference as transparent as possible and avoid technical

[7] If the variance of z is low then $(-x, x)$ is not an equilibrium for large x because a candidate can deviate to 0 and win with high probability. If $z \in [-\varepsilon, \varepsilon]$, for example, then equilibrium platforms cannot be more polarized than $\pm 2\varepsilon$, as deviating to the center would then guarantee victory.

[8] Specifying probabilistic voting this way, Bernhardt et al. (2009) show that candidate divergence can enhance social welfare, just as in the model below. Like polarization, the extent of this depends on variances, and hence on the extent of common interest: if z has low variance (relative to ε_i), the desired level of polarization is low; if it is high, common interest dominates voters' preferences.

[9] Policy motivated candidates only polarize in the first place if their intrinsic policy preferences are polarized. If voters know candidates' preferences, they should elect centrists, by the standard median voter logic. Anticipating this, extremists should be unlikely to run for office.

[10] As Footnote 2 points out, voters and candidates who are mostly selfish may become welfare motivated as large elections amplify altruism. Selfish candidates might also act welfare motivated to cultivate a favorable image or legacy.

[11] That is, $\ln f(z)$ is concave in z. Many of the most well-known distributions are log-concave (Bagnoli and Bergstrom, 2005).

[12] Quadratic utility is not essential but is standard, and is convenient because expected utility is maximized at the expectation of z, conditional on any available information.

[13] In this expression, $1_{w=j}$ is an indicator function that equals one if $w = j$ and zero otherwise.

[14] In McMurray (2017a) I emphasize how single-peaked preferences make voters risk averse, so that those with imprecise signals favor moderate policies, consistent with various empirical evidence. In line with those observations, Garz (2018) presents empirical evidence that information has a causal effect on voter polarization.

complexities that candidate signals introduce, this paper assumes that candidates possess *no* information of their own. This also underscores the strength of strategic inference, as candidates polarize *even* starting from identical policy beliefs.[15]

In the subgame associated with any pair $(x_A, x_B) \in X^2$ of candidate platforms, a voting strategy is a measurable function $v : S \to \Delta(\{A, B\})$ from the space of signals into the unit simplex over the set of candidates, with $v_j(s)$ specifying the probability of voting for candidate $j \in \{A, B\}$ in response to signal $s \in S$. Let V denote the set of such strategies. Abusing notation, let $v(s) = j$ indicate the pure strategy with $v_j(s) = 1$ and $v_{-j}(s) = 0$, where $-j$ is candidate j's opponent. When clear from context, let event j also denote the event $w = j$ of candidate j winning the election. When his peers all vote according to the strategy $v \in V$, a voter's best response $v^{br} \in V$ maximizes the expectation $E_{w,z}[u(x_w, z); v]$ of (1) for every private signal realization in S. A *(symmetric) Bayesian Nash equilibrium (BNE)* in the voting subgame is a strategy v^* that is its own best response.[16]

A voting strategy $\sigma : X^2 \to V$ in the complete game specifies voting behavior for every subgame. Let Σ denote the set of such strategies. $(x_A^*, x_B^*, \sigma^*) \in X^2 \times \Sigma$ is a *perfect Bayesian equilibrium (PBE)* if $\sigma^*(x_A, x_B)$ constitutes a BNE in the voting subgame associated with every platform pair $(x_A, x_B) \in X^2$ and the platform choice x_j^* of candidate $j \in \{A, B\}$ maximizes the expectation $E_{w,z}\left[u(x_w, z) + 1_{w=j}\beta; x_{-j}, \sigma\right]$ of (2), taking her opponent's platform x_{-j} and the voting strategy σ as given. Candidates are identical in this model, so for any PBE, there is another PBE with candidates reversing roles. For ease of exposition, focus on PBE that are *ordered* (*O*), meaning that $x_A^* \le x_B^*$. Given the symmetry of the model, it is also natural to focus on PBE that are *platform-symmetric* (*S*), meaning that $x_A^* = -x_B^*$. Let OSPBE denote a PBE that is both ordered and platform symmetric.

4. Equilibrium analysis

4.1. Voting subgame

For any platform pair $x_A < x_B$, the appendix shows that voting behavior is just as in McMurray (2017a), despite the more general signal structure here. Specifically, a voter favors the candidate whose platform is closest to his expectation $E(z|P, s_i)$ of the optimal policy, conditional both on his private signal s_i and on the *pivotal* event P of reversing the election outcome. This entails following a *threshold strategy* $v_t = \begin{cases} A & \text{if } s < t \\ B & \text{if } s > t \end{cases}$. There is a unique equilibrium threshold $t^*(\bar{x})$, which is the socially optimal threshold and which only depends on the midpoint $\bar{x} = \frac{x_A + x_B}{2}$ between the platforms.

Candidate A tends to win when z is low, while B wins when z is high. As Lemma 1 now states, Condorcet's jury theorem thus holds in this more general setting: the platform closest to z almost surely wins as n grows large.

Lemma 1 *(Jury Theorem). If $x_A \neq x_B$ then, for any realization of z, in the unique sequence of subgame equilibria, the candidate whose platform is closest to z wins with probability approaching* 1 *as n grows large.*

Lemma 1 is not surprising in light of jury theorem literature, but rests subtly on the well known *underdog effect* of the pivotal voting calculus, namely that one additional vote for the expected winner of an election is less likely to have impact than one vote for the expected loser. This mutes voters' policy beliefs, because even when his private signal is extreme, a voter reasons that z must lie between x_A and x_B, so the platform that seems worse must not be so bad. This muting of beliefs favors extreme candidates, who otherwise tend to look inferior.[17] In the following section, candidates respond to this by adopting more extreme platforms than they otherwise would.

4.2. Candidate best responses

This section assumes that voters follow the strategy $\sigma_{t^*} \in \Sigma$ that induces the equilibrium threshold strategy $v_{t^*(\bar{x})}$ whenever x_A and x_B have midpoint $\bar{x}$, and proceeds by backward induction to analyze candidates' platform incentives. Lemma 2 begins by stating the existence of a unique best response x_j^{br} to any opponent strategy $x_{-j} \in X$. If a candidate does not care about winning (i.e. $\beta = 0$), her best response is simply her expectation of z, conditional on any available information. Since F is symmetric and she observes no private signal, a candidate's expectation $E(z) = 0$ lies at the center of the policy space. However, a welfare motivated candidate *never* chooses this policy.

[15] In McMurray (2022) I show that candidates with high quality signals of their own still polarize, because the collective weight of so many voter signals still outweighs their own.

[16] With Poisson population uncertainty, BNE are necessarily symmetric across voters (Myerson, 1998).

[17] It may seem undesirable that pivotal considerations favor the candidate who is worse in expectation, but this is actually important for the jury theorem above. Suppose, for example, that $X = [-1, 1]$ and $(x_A, x_B) = (.8, 1)$, so that A and B are both far right of center. If they failed to take pivotal voting into account, the vast majority of voters would favor A, *even* in the rare case that $z > .9$, so that B is superior. The underdog effect favors B just enough that A still wins when $z < .9$, but candidates tie when $z = .9$ and B wins when $z > .9$. In this way, the underdog effect ensures that the proper candidate wins even in extreme states of the world.

Lemma 2. *If x_{-j} is left (right) of center then any best response x_j^{br} lies to the right (left) of x_{-j}. If $\beta = 0$ then x_j^{br} is unique and satisfies $x_j^{br} = E\left(z|w = j; x_j^{br}, x_{-j}, \sigma_{t^*}\right) > 0\ (< 0)$.*[18]

To see why the origin is not a best response, suppose that B wins the election when $x_A < x_B$. This will imply that most voters received high signal realizations, suggesting that z is high, so candidate B will update her expectation to $E(z|w = B) > 0$. In this way, the election outcome informs her about the private signals that voters observed. Intuitively, this information may seem useless, because she must commit to a policy position before learning whether she has won or lost the election; if she loses, she will infer opposite information. The key distinction is that a candidate's platform choice will affect her utility if she wins, but not if she loses. Thus, her best response is to behave in a way that will be optimal *if* it turns out that she has won the election, just as a voter optimally votes in a way that will be optimal *if* he is pivotal. In that sense, Lemma 2 highlights a pivotal inference akin to the standard voter calculus.[19]

4.3. Welfare motivated candidates

Building on Lemma 2, Theorem 1 states that a perfect Bayesian equilibrium exists. Given the symmetry of the model, candidates' equilibrium platforms can be symmetric around the origin. In fact, there is exactly one such equilibrium (up to a relabeling of the candidates). In large elections, the jury theorem reduces the game above to a simpler one in which candidates choose platforms and the one closer to z is implemented. If $x_A < x_B$ then candidates' expectations, conditional on winning, converge to $E(z|z < \bar{x})$ and $E(z|z > \bar{x})$. Theorem 1 does not rule out PBE with asymmetric platforms, but if these exist they become symmetric in the limit, as the log-concavity of f implies that $\bar{x} = 0$ is the only solution to the asymptotic requirement that $\frac{E(z|z<\bar{x})+E(z|z>\bar{x})}{2} = \bar{x}$.

Theorem 1. *For any n, there exists a unique OSPBE. If $\beta = 0$ then, for any sequence of PBE, $x^*_{A,n}$ and $x^*_{B,n}$ approach $E(z|z < 0)$ and $E(z|z > 0)$.*

If candidates adopted centrist platforms $(0, 0)$, voters would be indifferent, and might vote randomly. If they did, the pivotal calculus would reveal nothing about voters' signals, yielding identical posteriors $E(z|w = A) = E(z|w = B) = 0$. This might seem to validate $(0, 0)$ as an equilibrium platform pair, but it is not: deviating from $(0, 0)$ triggers a subgame with threshold voting, revealing voter information that validates the deviation. At least when $\beta = 0$, then, equilibrium platforms cannot coincide. In fact, when n is large, polarization is substantial: x^*_A and x^*_B lie at the means of opposite sides of a partition. Examples 1 through 3 offer stark examples of this. Such polarization is remarkable, given that candidates in this model are identical: their polarizing beliefs are purely endogenous.

Example 1 considers a uniform prior and voter signals drawn from a family of linear densities, which is the specification treated in McMurray (2017a). In equilibrium, candidates are more extreme than any voter. Example 2 considers another common information structure, namely a normal prior and normal signal, with correlation ρ. As long as ρ is not too high, voters are unlikely to favor policies privately that are more extreme than candidates' equilibrium policy positions.

Example 1 *(Uniform-linear model).* Let $z \sim U[-1, 1]$ and $g(s|z) = \frac{1}{2}(1 + sz)$ on $[-1, 1]$. Then $x^*_{A,n}$ and $x^*_{B,n}$ approach $\pm\frac{1}{2}$, the 25th and 75th percentiles of F, and voters' private expectations $E(z|s_i) = \frac{s_i}{3}$ range only from $-\frac{1}{3}$ to $\frac{1}{3}$, so no voter is as extreme as candidates.

Example 2 *(Normal-normal model).* Let $z \sim N(0, 1)$ and $s_i \sim N(0, 1)$ with $corr(s_i, z) = \rho$. Then $x^*_{A,n}$ and $x^*_{B,n}$ approach $\pm.8$, the 21st and 79th percentiles of F, and voter expectations $E(z|s_i) = \rho s_i \sim N\left(0, \rho^2\right)$ are over 75% likely to be less extreme than candidate platforms, provided $\rho < .69$.

In McMurray (2017a) I point out that many applications are best characterized by *binary truth*, meaning that one of two extremes is ultimately optimal. To exit an economic recession, for example, competing economic theories recommend either substantial economic stimulus or no stimulus at all. Intermediate levels of stimulus are better than embracing the wrong extreme, but are clearly not optimal, per se. Similarly, public funding can be split between two programs with the same purpose, such as raising teacher salaries or reducing class sizes to improve education, but if truth were known that one of these programs is actually more effective, it should ideally receive all available funding. Harrington (1993) proposes that deep philosophical attitudes toward the proper role of government may be binary, as well, as voters favor either "extensive or minimal government intervention in the economy."

[18] If $x_{-j} = 0$ then there are two best responses, x_j^{br} and $-x_j^{br}$.

[19] The proof of Lemma 2 is complicated by the fact that winning from a position to the right of x_{-j} suggests that z is high, but winning from a position to the left of x_{-j} suggests that z is low. However, if $x_{-j} < 0$ then a platform to the right of x_{-j} is more desirable than a platform below x_{-j}, as it improves utility in states where x_{-j} performs most poorly. Since a platform below x_{-j} is also likely to lose, it is even less attractive when $\beta > 0$.

With applications such as these in mind, and to make the underlying mechanisms as transparent as possible, Example 3 considers binary z. Candidates are maximally extreme, adopting -1 and 1 in large elections, even though interior policies are also feasible. Since G is assumed to be continuous, voter expectations $E(z|s)$ all lie between these extremes. Thus, candidates are more extreme in equilibrium than *any* voter.

Example 3 *(Binary truth model).* If $z \in \{-1, 1\}$ then $x^*_{A,n}$ and $x^*_{B,n}$ approach ± 1. For any G, no voter is as extreme as candidates.

4.4. Office motivated candidates

This section analyzes the case of $\beta > 0$, meaning that candidates not only care about the policy outcome, but also hope to win. Though entry decisions are not modeled here, this reflects the intuition that individuals who value office highly should be the most likely to campaign for office.

If candidates polarize, then since signals are continuous, there will be voters who favor policies between the candidates' platforms. By moving her own platform toward her opponent's, a candidate can attract some of these voters to her side. When β is low, a candidate foregoes the support of these voters to remain as close as possible to $E(z|w = j)$. As the importance of winning increases, polarization decreases, as Theorem 2 now states. If β is sufficiently large, the two candidates' platforms coincide, just as in standard median voter theorems. Theorem 2 is labeled the median *opinion* theorem because the voters attracted by centrist platforms are not those with centrist bliss points, but those with centrist signals of the common bliss point. As Section 4.5 emphasizes below, this has important consequences for social welfare.

Theorem 2 *(Median Opinion Theorem). There exists $\bar{\beta}$ such that, if (x^*_A, x^*_B, σ^*) is an OSPBE then $\beta \geq \bar{\beta}$ implies $x^*_A = x^*_B = 0$. Otherwise, $\left|x^*_j\right|$ decreases in β.*

Note that Theorem 2 holds even if truth is binary, as in Example 3. In that case, voters know that z lies either at -1 or 1, but because of imperfect signals, form expectations $E(z|s_i)$ strictly between -1 and 1. When a candidate moves toward the center, she attracts some of these voters away from her opponent, improving her chance of winning. In terms of the example above, a candidate who favors increasing teacher salaries pledges to use some of the available funding to reduce class sizes, appealing to voters who think teacher salaries are more important but lack confidence in their opinions. Similarly, a class size advocate pledges some funding for teacher salaries. When β is high enough, the teacher salary advocate promises half of available funding for class size reduction and the class size advocate pledges half of available funding for teacher salaries. In other words, competition for office drives them to compromise on the same centrist policy.

4.5. Welfare

Since the benefit to candidates of winning the election is zero-sum, while voters and candidates share common policy interests, it is uncontroversial to measure social welfare simply by the expectation of policy utility, (1). When $\beta = 0$, voter and candidate preferences are identical, so the logic of McLennan (1998) implies that socially optimal behavior is also individually optimal and can arise in equilibrium, as Proposition 1 now states.

Proposition 1. *For any n, there exists a strategy vector $\left(x^{**}_{A,n}, x^{**}_{B,n}, \sigma^{**}_n\right) \in X^2 \times \Sigma$ that maximizes expected welfare. If $\beta = 0$ then this vector also constitutes a PBE.*

Intuitively, the socially optimal policy position for a candidate with no information might seem to be $E(z) = 0$. However, a welfare motivated candidate's platform is optimal *given* that voters will be able to choose between the two platforms. Proposition 1 strengthens the logic of Condorcet's (1785) jury theorem: not only can voters choose the better of two platforms, candidates can produce an ideal menu of policies, even with no information of their own, and without explicitly coordinating. This also casts polarization in new light, since the behavior that is socially optimal is also polarized. Polarization has value precisely because it lets voters tailor their policy choice to the realized state of the world. This contrasts with standard private interest models, where convergence to the center maximizes welfare by keeping the policy outcome close to all voters' bliss points.[20]

If welfare motivated candidates polarize optimally, then by converging to the center, office motivated candidates underpolarize. Such candidates share voters' policy interests but recognize that even if truth is extreme, many voters will fail to realize it as such, and a centrist platform will attract these voters without estranging more extreme supporters. Depending on β, equilibrium policy platforms thus trade off welfare against the desire to win. Doing what is popular instead of what

[20] That the same empirical behavior can have opposite welfare implications underscores the importance of not taking the standard private interest paradigm for granted.

is right, and catering to voters' premature or uninformed opinions, is reminiscent of the binary pandering models listed in Section 1. However, the spatial model here offers richer geometry: *pandering* consists of remaining centrist, even when bold, extreme policy changes would be in voters' interest. As Section 1 notes, this can explain why extreme candidates are often lauded as confident visionaries while centrists are criticized for compromising their principles.

In some cases, the optimal level of polarization and the welfare loss from pandering may be substantial: if truth is binary, for example, then welfare motivated candidates champion policies at the opposite extremes of the policy space, but as explained in Section 4.4, competition for votes drives office motivated candidates toward the center. Strong enough office motivation thus produces compromises that are universally recognized *not* to be optimal, like splitting funding between superior and inferior programs, or issuing moderate sized economic stimulus that is certainly either much too large or much too small.

4.6. Large elections

Theorem 2 analyzes behavior for arbitrarily large β. However, extremely high values of β may not be relevant empirically. With quadratic policy utility, β compensates a candidate for a policy that is a distance $\sqrt{\beta}$ from z. When β is high, this distance may be large relative to the length of the policy interval. In the uniform-linear model above, for example, $\beta \geq 4$ makes a candidate willing to implement the *worst* policy in X, in order to win. $\beta \geq \frac{1}{4}$ makes her willing to tolerate policies a distance $\frac{1}{2}$ from z, which is 25% of the length of the policy space; given the widely perceived importance of public policy decisions, even β that high may be unrealistic. At the same time, public elections tend to be quite large. Instead of taking the limit as β grows large, fixing n, Theorem 3 therefore takes the limit as n grows large, fixing β.

Whatever platforms candidates adopt, random signals might lead voters to split evenly. Moving slightly closer to the center ensures that, when this happens, a candidate gets slightly more than half of the votes, not slightly less. When voters do not split evenly, though, slightly adjusting her platform doesn't affect whether she wins. As n grows large, the jury theorem guarantees that the superior candidate wins, so voters only split evenly if z happens to fall exactly between x_A and x_B. As her control over winning declines, a candidate becomes less willing (for any β) to sacrifice policy utility to moderate, and equilibrium polarization increases. Theorem 3 now states that, in the limit, the forces identified in Sections 4.3 and 4.4 both operate: polarization decreases in β, but $\bar{\beta}$ approaches $\frac{2E(z|z>0)}{f(0)}$, which is positive.

Theorem 3. *In the unique sequence of OSPBE, $x^*_{B,n}$ approaches* $\max\left\{0, E(z|z>0) - \frac{1}{2}\beta f(0)\right\}$ *and $x^*_{A,n}$ is symmetric.*

To illustrate the extent of polarization, Examples 4 and 5 recompute platform positions for the policy environments of Examples 1 and 2, now with $\beta = \frac{1}{4}$ instead of $\beta = 0$. As noted above, this compensates a candidate for policies that differ from z by a distance of $\frac{1}{2}$. In the uniform-linear model, this distance is 25% of the length of the feasible policy interval. In that case, polarization proves to be quite robust: office motivated candidates are only slightly less extreme than welfare motivated candidates (the 28th/72nd percentiles of F, in from the 25th/75th), and are still more extreme than any voter. Platform convergence requires that $\beta \geq 2$, meaning that a candidate is willing to commit to the *worst* policy in order to win.

Example 4. In the uniform-linear model, if $\beta = \frac{1}{4}$ then $x^*_{A,n}$ and $x^*_{B,n}$ approach $\pm.44$, the 28th and 72nd percentiles of F, and no voter is as extreme as candidates. Platforms only coincide if $\beta \geq 2$.

In the normal-normal model, z is 68% likely to lie in $[-1, 1]$, so reasonable levels of β are similar to the uniform case. If $\beta = \frac{1}{4}$ then platforms become less extreme, but only slightly so: the 23rd and 77th percentiles of F, in from the 21st and 79th. Even if $\beta = 1$, so that winning compensates for a full unit of policy distance, platforms are as polarized as the 27th and 73rd percentiles of F. In Example 2, voter expectations were over 75% likely to be less extreme than candidate platforms, provided $\rho < .69$. With $\beta = \frac{1}{4}$ or $\beta = 1$, this bound on ρ decreases only to .65 or .52. Given the inherent noisiness of voter opinions, correlations this low seem highly plausible. Platforms only converge in this example if $\beta \geq 4$, which compensates for a policy distance of 2.

Example 5. In the normal-normal model, if $\beta = \frac{1}{4}$ ($\beta = 1$) then $x^*_{A,n}$ and $x^*_{B,n}$ approach $\pm.75$ ($\pm.6$), the 23rd (27th) and 77th (73rd) percentiles of F, and voter expectations $E(z|s_i)$ are over 75% likely to be less extreme than candidates' platforms, provided $\rho < .65$ ($\rho < .52$). Platforms coincide only if $\beta \geq 4$.

If truth is binary, as in Example 3, then polarization turns out to be perfectly robust, as Example 6 now states: candidates polarize to opposite extremes of the policy space for arbitrarily large values of β, just as they do for $\beta = 0$. It remains true that a moderate platform would win more votes, and when $\beta \geq 4$, a candidate is willing to promise *any* policy in order to win. But in a large election, compromise is simply unnecessary: if truth is on her side then a candidate will win even from a polarized position; if truth is against her, she will lose even if she moderates.

Example 6. If $z \in \{-1, 1\}$ then $x^*_{A,n}$ and $x^*_{B,n}$ approach ± 1 for any β. If G is continuous, no voter is as extreme as candidates.

Taken together, Examples 4 through 6 make clear that, for a variety of specifications, polarization remains substantial in large elections, even when β is high. In that sense, polarization is the robust prediction of a common-interest model, just as convergence is robust in private interest settings. That also means that the menu of policies voters is left to choose between should be close to ideal, despite candidates' office motives.

5. Conclusion

From the perspective of standard private-interest election models, polarization is both perplexing and disturbing, representing some inexplicable democratic failure. A spatial version of Condorcet's (1785) common interest paradigm casts polarization in new light: centrist policies still give candidates a competitive advantage but reflect pandering to uninformed voters, not utilitarian compromise. This advantage is only slight, so candidates may polarize despite strong desires to win. Ironically, polarization stems from underlying unity: a candidate trusts voters to support an extreme platform if it is truly superior. Substantial polarization can also be optimal for voters.

In McMurray (2021) I show that the analysis above extends readily to multiple dimensions, which is a well known limitation of private interest models.[21] In McMurray (2022) I show that candidates still make pivotal inference from voters and polarize, even when their own private signals are more reliable than voters', because majority opinion is infallible and overwhelms the information content of even very precise candidate signals. Important directions for future exploration include information frictions such that large electorates still make mistakes, and conflicts of interest among or between voters and candidates.[22] If this makes candidates infer less from voters, they might polarize less and rely more on their own private policy opinions. As long as swings in public opinion occur for reasons that policy positioning cannot control, however, polarization seems likely to remain robust.

In the model above, candidates can be too centrist but never too polarized, and the optimal level of polarization can be quite high. In practice, though, vote shares tend to be close to 50%; with similar public support, $E(z|w=A)$ and $E(z|w=B)$ should not differ substantially from $E(z)$. In that light, substantial empirical polarization still seems excessive, even through the lens of a common interest model. Exploring this is an important direction for future work. Refusal to compromise might reflect overconfident policy beliefs or catering to overconfident voters.[23] Candidates might also overpolarize to project confidence and competence.[24] Ultimately, polarization may be more damaging with common than with private interests, especially in extensions where electorates are prone to mistakes: not only do the losses to voters who endure their least favorite policies exceed the gains to voters who get what they want, as in private interest settings; overextremism might produce policies that in truth are bad for everyone.

Appendix A

Lemma 3. *If $x_A < x_B$ then the unique best response to $v \in V$ in the (x_A, x_B) subgame is the threshold strategy $v_{t^{br}}$, where t^{br} solves $E\left(z|P, s_i = t^{br}; v\right) = \bar{x} = \frac{x_A + x_B}{2}$.*

Proof of Lemma 3. If voters follow $v \in V$ then, for $z \in Z$, each votes for candidate $j \in \{A, B\}$ with the following probability.

$$\phi(j|z; v) = \int_S v_j(s)\, g(s|z)\, ds \tag{A.1}$$

In terms of (A.1), the numbers N_A and N_B of A votes and B votes are Poisson random variables with means $n\phi(A|z)$ and $n\phi(B|z)$ (suppressing the dependence on v), so the joint distribution of $N_A = a$ and $N_B = b$ is simply the product $\psi(a, b|z) = \frac{e^{-n\phi(A|z) - n\phi(B|z)}}{a!b!} [n\phi(A|z)]^a [n\phi(B|z)]^b$ of Poisson probabilities. Candidate j wins the election (event j) with probability $\Pr(j|z) = \Pr\left(N_j > N_{j'}|z\right) + \frac{1}{2}\Pr\left(N_j = N_{j'}|z\right)$ in state z, where the second term reflects the possibility of winning a tie-breaking coin toss.

As in Myerson (1998), a voter within the game reinterprets N_A and N_B as the numbers of votes cast by his peers; by voting himself, he can add one to either total, thus increasing $\Pr(j|z)$ by $\Pr\left(P_j|z\right) = \frac{1}{2}\Pr\left(N_j = N_{j'}|z\right) + \frac{1}{2}\Pr\left(N_{j'} = N_j + 1|z\right)$, which is the probability that his j vote is pivotal (event P_j) because the candidates had tied and lost the tie-breaking coin toss or because j had trailed by exactly one vote but would have won the tie breaker. The difference in expected utility between voting A and voting B can therefore be written in terms of the event $P = P_A \cup P_B$ of being pivotal, as follows,

[21] This can also explain why logically related issue positions are bundled together so consistently, creating an illusion of unidimensionality.

[22] Feddersen and Pesendorfer (1997) derive a jury theorem for voters with a mix of private and common interests, so welfare motivated candidates would still polarize in that case, although private interest also polarizes voters, as Section 2 points out, making candidates less polarized by comparison..

[23] Empirically, primary election voters tend to hold extreme policy beliefs and be confident of winning (McMurray, 2017a), and tend to favor extreme candidates (Brady et al., 2007; Hall and Snyder, 2015).

[24] This occurs in Kartik et al. (2015), where voters infer the location of z from candidates' policy platforms, and might also occur if voters use confidence on current issues to forecast candidates' competence handling future issues.

$$\Delta_{AB}(s_i) = E_z\{[u(x_B,z) - u(x_A,z)]\Pr(P_B|z)|s_i\}$$
$$-E_z\{[u(x_A,z) - u(x_B,z)]\Pr(P_A|z)|s_i\}$$
$$= 2(x_B - x_A)\Pr(P|s_i)[E(z|P,s_i) - \bar{x}] \tag{A.2}$$

where the last equality follows as in McMurray (2017a). $E(z|P,s_i)$ increases in s_i (by the MLRP of G), so there exists a threshold t^{br} such that $E\left(z|P,s_i = t^{br}\right) = \bar{x}$, and a voter prefers to vote B if and only if s_i exceeds t^{br}. □

Lemma 4. *There exists a unique function $t^*: X \to S$ such that, for any $x_A \leq x_B$ with midpoint $\bar{x} \in X$, the threshold strategy $v_{t^*(\bar{x})}$ constitutes a BNE in the (x_A, x_B) subgame and is socially optimal in V. If $x_A < x_B$ then $v_{t^*(\bar{x})}$ is the only BNE and the only socially optimal strategy. t^* increases in $\bar{x}$ (and therefore in x_A and x_B) and satisfies $t^*(-\bar{x}) = -t^*(\bar{x})$.*

Proof of Lemma 4. For a threshold strategy v_t, (A.1) reduces to $\phi(A|z) = G(t|z)$ and $\phi(B|z) = 1 - G(t|z)$. Since G satisfies MLRP, these decrease and increase in z, respectively. As I show in the proof of Proposition 1 of McMurray (2017a), this implies that increasing t shifts the distribution of P upward in the sense of first-order stochastic dominance, so that $E(z|P,s_i)$ increases. The solution t^{br} to $\Delta_{AB}(s_i; v_t) = 0$ therefore decreases in t, implying the existence of a unique fixed point $t^* = t^{br}(t^*)$, and v_{t^*} is therefore the unique BNE voting strategy when $x_A < x_B$. (When $x_A = x_B$, any v constitutes a BNE.) Fixing $x_B - x_A$ and increasing $\bar{x}$ decreases $\Delta_{AB}(s_i; v_t, \bar{x})$ for any t. Since this expression increases in t, the solution $t^{br}(t;\bar{x})$ to $\Delta_{AB}(s_i; v_t, \bar{x}) = 0$ increases in $\bar{x}$. Since $t^{br}(t;\bar{x})$ decreases in t and increases in $\bar{x}$, the fixed point $t^*(\bar{x}) = t^{br}(t^*;\bar{x})$ increases in $\bar{x}$.

Under the narrow topology, expected utility $E_{w,z}(u;v) = \int_Z \sum_{w=A,B}\Pr(w|z;v)\,u(x_w,z)\,f(z)\,dz$ is continuous in v over the compact space V (Balder, 1988), so an optimal $v^* \in V$ exists by the Weierstrass extreme value theorem. By the logic of McLennan (1998), any such strategy also constitutes a BNE. The uniqueness of BNE then implies that v^* is also uniquely optimal. Symmetry holds because $\phi(A|z; v_{-t}) = G(-t|z) = 1 - G(t|-z) = \phi(B|-z; v_t)$ for any t, so $\psi(a,b|z; v_{-t}) = \psi(b,a|-z; v_t)$, $\Pr(P_A|z; v_{-t}) = \Pr(P_B|-z; v_t)$, $\Pr(P|s_i; v_{-t}) = \Pr(P|-s_i; v_t)$, and $E(z|P,s_i,v_{-t}) = -E(z|P,-s_i,v_t)$, implying from (A.2) that (fixing $x_B - x_A$) $\Delta_{AB}(s_i; v_{-t}, -\bar{x}) = -\Delta_{AB}(-s_i; v_t, \bar{x})$ and therefore that $t^{br}(t;-\bar{x}) = -t^{br}(-t;\bar{x})$, so $t^* = t^{br}(t^*;\bar{x})$ implies that $t^{br}(-t^*;-\bar{x}) = -t^{br}(t^*;\bar{x}) = -t^*$. □

Proof of Lemma 1. The proof of this result is essentially the same as that of Proposition 3 of McMurray (2017a), but with equilibrium characterized by Lemma 4 and with the monotonicity of $\phi(A|z;v_t) = G(t|z)$ and $\phi(B|z;v_t) = 1 - G(t|z)$ owing to the assumption that $G(s|z)$ satisfies MLRP. □

Proof of Lemma 2. Taking σ_{t^*} as given, the expectation of $u_j = u(x_w,z) + \beta 1_{w=j}$ can be written as follows.

$$E_{w,z}\left(u_j; x_A, x_B\right) = E_z\left[\sum_{w=j,-j} u(x_w,z)\Pr(w|z)\right] + \beta\Pr(w=j) \tag{A.3}$$

To simplify the analysis, focus first on the case of $\beta = 0$, so that candidates do not value winning per se. In that case, differentiating (A.3) yields the following,

$$\frac{\partial E_{w,z}\left(u_j; x_A, x_B\right)}{\partial x_B} = E_z[2(z - x_B)\Pr(B|z)] + \frac{1}{2}\frac{\partial E_{w,z}[u(x_w,z); v_{t^*}]}{\partial t^*(\bar{x})}\frac{\partial t^*(\bar{x})}{\partial \bar{x}}$$
$$= 2\Pr(B)[E(z|B;\bar{x}) - x_B] \tag{A.4}$$

where the second equality holds because optimal voting (Lemma 4) implies that $\frac{\partial E_{w,z}[u(x_w,z);v_{t^*}]}{\partial t^*(\bar{x})} = 0$. (A.4) is positive for $x_B = -1$, negative for x_B just below x_A, positive just above x_A, and negative for $x_B = 1$, implying that (A.3) is "M-shaped" in x_B, with local minima at -1, x_A, and 1, and local maxima $x^L \in (-1, x_A)$ and $x^R \in (x_A, 1)$, which both solve $E(z|B; x_A, x_B) = x_B$. Moreover, $x^L < 0 < x^R$, since $x_A < x_B$ implies that $F(z|B)$ first-order stochastically dominates $F(z)$, so $E(z|B) > 0$, while $x_A > x_B$ implies that $E(z|B) < 0$.

Starting from x_A, expected utility increases more quickly moving toward the origin than away from it. To see this, first let $x_A < 0$ and consider platforms $x_A - 2\varepsilon$ and $x_A + 2\varepsilon$ that are left and right of x_A by a distance $2\varepsilon < \frac{x_A - x^L}{2}$. By symmetry, the expected utility of $(x_A, x_A - 2\varepsilon)$ is the same as that of $(x_A - 2\varepsilon, x_A)$, which can be written as $(\bar{x} - \varepsilon, \bar{x} + \varepsilon)$ for $\bar{x} = x_A - \varepsilon$. $(x_A, x_A + 2\varepsilon)$ can be written as $(\bar{x} - \varepsilon, \bar{x} + \varepsilon)$, as well, but for $\bar{x} = x_A + \varepsilon$. Thus, moving from one to the other simply amounts to increasing $\bar{x}$ by 2ε. Writing (A.3) as follows

$$E_{w,z}\left(u_j; \bar{x} - \varepsilon, \bar{x} + \varepsilon\right) = E_z[u(\bar{x} - \varepsilon, z)\Pr(A|z;\bar{x}) + u(\bar{x} + \varepsilon, z)\Pr(B|z;\bar{x})]$$

and differentiating with respect to $\bar{x}$ yields the following, since t^* is chosen optimally by voters.

$$\frac{\partial}{\partial \bar{x}} E_{w,z}\left(u_j; x_A, x_B\right) = E_z\left[-2\left(\bar{x} - \varepsilon - z\right)\Pr\left(A|z;\bar{x}\right) - 2\left(\bar{x} + \varepsilon - z\right)\Pr\left(B|z;\bar{x}\right)\right] + \frac{\partial E_{w,z}\left[u\left(x_w, z\right); \bar{x}, \sigma_{t^*}\right]}{\partial t^*\left(\bar{x}\right)}$$
$$= -2\bar{x} + 2\varepsilon\left[\Pr\left(A;\bar{x}\right) - \Pr\left(B;\bar{x}\right)\right]$$

This is positive when $\bar{x} < 0$. Integrating over $\bar{x}$ from $x_A - \varepsilon$ to $x_A + \varepsilon$ then yields the total gain in expected utility, which must be positive as well.[25] That this holds for $x_B = x^L$ implies that $E_{w,z}\left(u_j; x_A, x_B = x^L\right) < E_{w,z}\left(u_j; x_A, x_B = x_A + \left(x_A - x^L\right)\right) \le \max_{x_B > x_A} E_{w,z}\left(u_j; x_A, x_B\right) = E_{w,z}\left(u_j; x_A, x_B = x^R\right)$. Thus, x^R is the global optimum. Symmetrically, x^L is the global optimum when $x_B > 0$. Thus, when $\beta = 0$, there is a unique best response $x_j^{br} \in \left\{x^L, x^R\right\}$ solving $E\left(z|B; x_A, x_B\right) = x_B$.

Away from $x_B = x_A$, differentiating $\Pr\left(B;\bar{x}\right)$ with respect to x_B yields the following,

$$\frac{\partial \Pr\left(B;\bar{x}\right)}{\partial x_B} = \frac{1}{2}\frac{\partial \Pr\left(B\right)}{\partial \phi\left(B\right)}\frac{\partial \phi\left(B\right)}{\partial t^*\left(\bar{x}\right)}\frac{\partial t^*\left(\bar{x}\right)}{\partial \bar{x}}$$

which is negative since $\frac{\partial \Pr(B)}{\partial \phi(B)}$ and $\frac{\partial t^*(\bar{x})}{\partial \bar{x}}$ are positive and $\frac{\partial \phi(B)}{\partial t^*(\bar{x})}$ is negative. For $x_B > x^R$, therefore, the derivative of (A.3) is unambiguously negative, implying that x_B is not a best response to x^R. Similar reasoning rules out a best response below x^L. Since expected utility is continuous in x_B over the compact interval $\left[x^L, x^R\right]$, a best response exists by the extreme value theorem. However, if $x_A < 0$ then $x_B = x_A - \varepsilon$ does not constitute a best response for $\varepsilon > 0$, as it is dominated by $x_B = x_A + \varepsilon$, which is closer to the origin, and so generates higher policy utility, as shown above, and also generates a higher win probability, as $\Pr\left(B; x_A, x_A - \varepsilon\right) < \frac{1}{2} < \Pr\left(B; x_A, x_A + \varepsilon\right)$. □

Proof of Theorem 1. If $\left(x_A, x_B\right) = \left(-x, x\right)$ then $\bar{x} = 0$, so by Lemma 4, voters' equilibrium response is a threshold strategy with $t^*\left(0\right) = 0$. This generates symmetric voting outcomes $\phi\left(A|z\right) = \phi\left(B|-z\right)$, $\Pr\left(A|z\right) = \Pr\left(B|-z\right)$, $\Pr\left(A\right) = \Pr\left(B\right) = \frac{1}{2}$, and $E\left(z|A\right) = -E\left(z|B\right)$. The derivative of (A.3) then reduces from (A.4) simply to $\frac{\partial E_{w,z}\left(u_B; x_A = -x, x_B = x\right)}{\partial x_B} = E\left(z|B; \bar{x} = 0\right) - x$. Symmetrically, $\frac{\partial E\left(u_A; x_A = -x, x_B = x\right)}{\partial x_A} = E\left(z|A; \bar{x} = 0\right) - \left(-x\right) = x - E\left(z|B; \bar{x} = 0\right)$. Therefore, $E_{w,z}\left(u_A; x_A, x_B = x_n^*; n\right)$ and $E_{w,z}\left(u_B; x_A = -x_n^*, x_B, n\right)$ are maximized at $x_A = -x_n^*$ and $x_B = x_n^*$ if and only if $x_n^* = E\left(z|B; \bar{x} = 0, n\right)$, and $\left(-x_n^*, x_n^*, \sigma_n^*\right)$ is the unique OSPBE for population parameter n.

Lemma 1 implies that $\lim_{n\to\infty} \Pr\left(B\right) = 1 - F\left(\bar{x}\right)$, $\lim_{n\to\infty} E\left(z|B\right) = E\left(z|z > \bar{x}\right)$, and $\lim_{n\to\infty} \frac{\partial \Pr(B)}{\partial \bar{x}} = -f\left(\bar{x}\right)$, so for any sequence $\{x_{Bn}\}$ of platforms, the derivative of (A.3) approaches the following limit.

$$\lim_{n\to\infty} \frac{\partial E\left(u_B\right)}{\partial x_B} = 2\left[1 - F\left(\bar{x}\right)\right]\left[E\left(z|z > \bar{x}\right) - \lim_{n\to\infty} x_{B,n}\right] - \frac{1}{2}\beta f\left(\bar{x}\right) \tag{A.5}$$

If $\left(x'_{A,n}, x'_{B,n}\right)$ are PBE platforms then $\frac{\partial E(u_B)}{\partial x_B} = 0$ for any n, so $\lim_{n\to\infty} \frac{\partial E(u_B)}{\partial x_B} = 0$ as well. From (A.5), this requires $\lim_{n\to\infty} x'_{B,n} = E\left(z|z > \bar{x}\right)$ when $\beta = 0$. A corresponding condition requires that $\lim_{n\to\infty} x'_{A,n} = E\left(z|z < \bar{x}\right)$. Together with the definition of $\bar{x}$, then, it must be that $\frac{E(z|z<\bar{x}) + E(z|z>\bar{x})}{2} = \bar{x}$. It is straightforward to show that this holds for $\bar{x} = 0$, given the symmetry of F. No other solution exists, as the log-concavity of f implies that $E\left(z|z < \bar{x}\right)$ and $E\left(z|z > \bar{x}\right)$ both increase in $\bar{x}$ with slope smaller than one (Bagnoli and Bergstrom, 2005). □

Proof of Theorem 2. For any symmetric platform pair $\left(x_A, x_B\right) = \left(-x, x\right)$, $\bar{x} = t^*\left(\bar{x}\right) = 0$. Other than x_B, the terms in (A.4) are the same for any such pair. Since $\frac{\partial \Pr(B)}{\partial \phi(B)} > 0$, $\frac{\partial \phi(B)}{\partial t^*} < 0$, and $\frac{\partial t^*(\bar{x})}{\partial \bar{x}} > 0$ (by Lemma 4), (A.4) decreases linearly in β and is negative for any $\beta > \bar{\beta} = \frac{4\Pr(B)\left[x_B - E(z|B)\right]}{\frac{\partial \Pr(B)}{\partial \phi(B)}\frac{\partial \phi(B)}{\partial t^*}\frac{\partial t^*(0)}{\partial \bar{x}}}$. In that case, the only OSPBE is $\left(x_A^*, x_B^*\right) = \left(0, 0\right)$. For $\beta < \bar{\beta}$, the derivative of (A.3) is zero if and only if $x_B == -x_A = E\left(z|B\right) + \frac{1}{2}\beta\frac{\partial \Pr(B)}{\partial \phi(B)}\frac{\partial \phi(B)}{\partial t^*}\frac{\partial t^*(0)}{\partial \bar{x}}$. The final product in this expression is negative, so $x_B^*\left(\beta\right)$ decreases in β in that case. □

Proof of Proposition 1. Lemma 4 states that the equilibrium voting strategy $\sigma_n^{**} = \sigma_{t^*}$ optimally responds to any pair $\left(x_A, x_B\right) \in X^2$ of candidate platforms, so a search for an optimal $\left(x_{A,n}^{**}, x_{B,n}^{**}, \sigma_n^{**}\right)$ reduces to a search for optimal candidate platforms $\left(x_{A,n}^{**}, x_{B,n}^{**}\right)$ that can be coupled with σ_{t^*}. Similarly, when $\beta = 0$, candidate utility is identical to social welfare, so a best response characterizes the social optimum for one candidate, holding the other's platform and voter behavior fixed. $F\left(z|z > \bar{x}\right)$ first-order stochastically dominates $F\left(z|w = B; \bar{x}\right)$, which first-order stochastically dominates $F\left(z\right)$, so $0 = E\left(z\right) < E\left(z|B; \bar{x}\right) < E\left(z|z > \bar{x}\right)$. When x_A and x_B are both best responses, therefore, $E\left(z|B\right) < E\left(z|z > \bar{x}\right) < E\left(z|z > \frac{0 + x_B}{2}\right)$. The log-concavity of f implies that $\frac{d}{dx_B} E\left(z|z > \frac{x_B}{2}\right) = \frac{1}{2}E\left(z|z > x_B\right) < \frac{1}{2}$, so there exists $\hat{x}_B > 0$ such that if $x_B > \hat{x}$ then $x_B^{br} < E\left(z|z > \frac{x_B}{2}\right) < x_B$. Symmetrically, if $x_A < -\hat{x}$ then $x_A^{br} > E\left(z|z < \frac{x_A}{2}\right) > x_A$. Thus, (A.4) is positive for $x_A < -\hat{x}$, implying

[25] This is trivial if $\varepsilon < |x_A|$. Otherwise, it relies on symmetry, as $\frac{\partial}{\partial \bar{x}} E_{w,z}\left(u_j; -\bar{x}\right) = -\frac{\partial}{\partial \bar{x}} E_{w,z}\left(u_j; \bar{x}\right)$, so $\int_{x_A-\varepsilon}^{x_A+\varepsilon} \frac{\partial}{\partial \bar{x}} E_{w,z}\left(u_j; x_A, x_B\right) d\bar{x} = \int_{x_A-\varepsilon}^{-x_A-\varepsilon} \frac{\partial}{\partial \bar{x}} E_{w,z}\left(u_j; x_A, x_B\right) d\bar{x} > 0$.

that $\left(-\hat{x}, x_B\right)$ provides higher welfare than (x_A, x_B). Similarly, if $x_B > \hat{x}$ then $\left(x_A, \hat{x}\right)$ provides higher welfare than (x_A, x_B). Together, this means that (x_A, x_B) is not optimal unless $(x_A, x_B) \in \left[-\hat{x}, \hat{x}\right]^2$. Expected utility is continuous over this compact interval, so an optimum $\left(x_A^{**}, x_B^{**}\right)$ exists by the extreme value theorem. By the logic of McLennan (1998), this (together with σ^*) also constitutes a PBE when $\beta = 0$. □

Proof of Theorem 3. As n grows large, $\frac{\partial E(u_B)}{\partial x_B}$ approaches (A.5). For an OSPBE, $\bar{x} = 0$, so this reduces to the following,

$$\lim_{n \to \infty} \frac{\partial E\left(u_B\right)}{\partial x_B} = \left[E\left(z|z > 0\right) - x_B\right] - \frac{1}{2}\beta f\left(0\right)$$

which is negative for all x_B if $E\left(z|z > 0\right) - \frac{1}{2}\beta f\left(0\right) \leq 0$, implying that neither candidate can improve on the platform pair $(0, 0)$. Otherwise, solutions to the first-order equilibrium conditions must approach the solution to $\lim_{n \to \infty} \frac{\partial E(u_B)}{\partial x_B} = 0$, so $x_{B,n}^*$ approach $E\left(z|z > 0\right) - \frac{1}{2}\beta f\left(0\right)$ (and $x_{A,n}^*$ is symmetric). □

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