Candidate Opinions versus Voter Opinions

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Abstract

This paper analyzes a spatial common interest election in which voters and candidates both observe private signals of the location of an optimal policy. A surprisingly complex interaction between candidates' platform decisions and strategic inference of others' information can produce inefficient and unusual equilibria, such as one candidate overreacting and the other underreacting to their private information. The most robust equilibrium seems to be that candidates polarize, effectively ignoring their own private policy opinions. For example, a candidate who believes the far left to be optimal may commit to a policy on the right, or vice versa.

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1 Introduction

Many of society's most important eventual policy goals (e.g. peace, prosperity, and economic stability) are universally valued but perplexing to achieve. In analyzing the political process, imperfect information is thus an essential ingredient. Finding answers to perplexing policy issues is ultimately a collaborative effort: candidates must first identify good policies from the myriad of possibilities and adopt these as platforms, then voters must determine which candidate's platform policy is most likely to accomplish their shared goals.

As summarized below, existing election models exclude either candidate information or voter information, or include both, but only in a binary policy setting. The first contribution of this paper is simply to introduce and explore a model in which candidates and voters all have private signals of what should be done, in a spatial setting where there are a continuum of policies to choose from (any of which might be optimal).[1]

It may seem relatively straightforward that $n + 2$ individuals with a shared objective should be able to pool their information effectively. The second contribution of this paper is simply to alert future modelers

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[1]Information is most important when voters and candidates share a common objective. An eventual direction of interest would be to accommodate private interests, as well, but as a first step, the model below assumes that voter and candidate interests coincide exactly. As I point out in McMurray (2013), this could also be realistic, if large elections amplify otherwise low levels of altruism.

that this apparent simplicity is deceiving. One layer of complexity is well known: a strategic voter partly infers other voters' information from the presumed event of a pivotal vote; here, he also utilizes any information revealed by candidates' platform positioning decisions. McMurray (2022) introduces a second layer of complexity, which is that candidates, too, have a pivotal calculus of sorts, inferring some voter information (even before votes are cast) from the presumed event of winning the election; here, a candidate modifies this inference because voter behavior partly reflects voters' private information, but now also duplicates her own information, revealed by her platform choice. A new, but still mild layer of complexity is that, from any platform position that her opponent might adopt, a strategic candidate should also infer her opponent's private information.

A fourth layer of complexity, which is both new and more challenging than the rest, is that the message communicated by voter behavior depends on the location of a candidate's policy platform *relative* to her opponent's; winning the election from just left of her opponent's platform conveys very different information than winning from just to the right. Her opponent will adopt different platforms for every signal realization, so a candidate's expected utility is kinked in many places, and highly non-monotonic. This makes it difficult to confirm any local best response as a global best response, and therefore difficult to establish any behavior as an equilibrium.

Intuitively, candidates have a harder information problem than voters, since they need to identify good platforms from a continuum of possibilities; voters need only determine which candidate's platform is best. Candidates are likely to have much better information than the typical voter, as well, since they have career incentives to learn about public policy, may have privileged access to (e.g. classified) information, and are the natural targets of informational lobbying. For these reasons and because voters' impact depends first on candidates', it may seeem intuitive that candidate information is the key ingredient for good outcomes overall. Contrary to this intuition, however, the third contribution of this paper is to show that, at least under canonical signal assumptions, voter information matters much more than candidates'. In fact, candidate information may play almost no role in determining policy. Specifically, candidate signals in the model below may be much higher quality than voter signals, but candidates may behave in equilibrium almost as if they held no policy opinions of their own. For example, a candidate may commit to a policy left of center when she privately believes the best policy is far right, or vice versa.

The simple logic behind this result is Condorcet's "jury" theorem, which points out that large numbers of poorly informed voters can be collectively better informed than a single expert, such as a political candidate. Even if her own signal is much more reliable than a voter's, a candidate recognizes that voters collectively are better informed than she is: under canonical information assumptions, they are virtually infallible.[2] If her own opinion differs from voters' consensus, it must be mistaken. Thus, she is willing to commit to a policy even opposite her private signal.

The complexities above limit the extent to which a general version of the model below can be analyzed,

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[2]The jury theorem states that, in common interest settings with sufficiently informative voter signals, an increasingly large electorate chooses the better of two alternatives with probability approaching one (Black, 1958).

but for a specific distribution of signals, the model can be analyzed numerically. The fourth contribution of the paper is to show that the non-monotonicity described above can produce multiple equilibria. These may be inefficient, and include unusual behavior. In one equilibrium, for example, one candidate overreacts to her private information while the other underreacts to hers.

Even numerical analysis is difficult unless the number of signal realizations is quite small. The fifth contribution of this paper is to rule out some equilibrium configurations for tractable instances of the model, concluding that the most robust equilibrium outcome seems to be polarization, just as in the model with no candidate signals: one candidate takes a position on the left and the other takes a position on the right. This can be inefficient, but seems in general to be efficient.

Related Literature Most common-interest election models focus only on voters, with candidate policy positions fixed exogenously. McMurray (2022) explores how candidates' policy positioning reacts to voter information, but candidates in that paper possess no signals of their own. Kartik, Squintani, and Tinn (2024) study how policy positioning reflects candidates' private information, but in a model with no voter signals. Of course, understanding the relative importance of voter versus candidate information requires a model with *both*.

Loertscher (2012) combines voter and candidate signals, but in binary decisions which lack the richness of the spatial policy choices studied below. This and the related binary models of Heidhues and Lagerlöf (2003), Laslier and Van de Straeten (2012), Gratton (2014), and Klumpp (2014) also assume that candidates are office motivated, focusing on the extent to which policy positioning reveals candidates' private information to voters. In contrast, the model below takes seriously that voters and candidates effectively collaborate on the same difficult policy decisions.

The result that the "pivotal" event of winning the election dramatically shapes candidates' strategic incentives extends a theme from voting literature, which repeatedly finds that pivotal inference dramatically alter voters' strategic incentives. In Feddersen and Pesendorfer (1996) and McMurray (2013), for example, some voters abstain from voting to avoid the pivotal event of contradicting an informed majority. In Feddersen and Pesendorfer (1998), jurors may unanimously convict a defendant that they all privately believe is innocent, each reasoning that if his own vote is pivotal, other jurors must all have voted guilty. In Tajika (2022), voters all vote for the policy that seems *worst*, knowing that ties are most likely when the evidence is most deceptive.[3]

2 The Model

Players and payoffs A game is played as follows. First, candidates A and B simultaneously choose policy platforms x_A and x_B within a closed interval X of policy alternatives. Next, N voters simultaneously each

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[3]Outside the common interest literature, the pivotal voting calculus determines voters' willingness to pay voting costs (Riker and Ordeshook, 1968) and to vote strategically for candidates who are less preferred but more likely to win (Duverger, 1957; Myerson and Weber, 1993).

vote for one of the two candidates, where N follows a Poisson distribution with mean n, as in Myerson (1998). The candidate $w \in \{A, B\}$ who receives more votes (breaking a tie, if necessary, with equal probability) wins the election and implements her platform x_w. Voters and candidates then each receive the following policy utility,

$$u(x_w, z) = -(x_w - z)^2 \tag{1}$$

which decreases quadratically in the distance from some policy $z \in X$ that nature determines is superior to any other.

Private information The location of the optimal policy z is unknown, but follows a known prior F with density f. Before the game, candidates and voters also observe private signals that are informative of z. Candidate signals s_j are drawn from a finite ordered set $S \equiv \{s_1, s_2, ..., s_K\}$, according to a distribution G with mass function g. Voter signals s_i are drawn from an interval $I \subseteq \mathbb{R}$ according to a known distribution H with density h. Conditional on z, all voter and candidate signals are independent. That $G \neq H$ is not essential for the analysis below, but allows the possibility that candidates are better informed about policy issues than a typical voter is.[4] Assume that G and H both satisfy the strict monotone likelihood ratio property (MLRP), which ensures that higher signal realizations are more likely in higher states, so that candidate and voter signals are informative of z.[5]

Strategies Abusing notation, let $j \in \{A, B\}$ denote both a candidate (with opponent $-j$) and the event $w = j$ of that candidate winning the election. Abusing notation again, let x_j denote both a specific policy position (in X) adopted by candidate j and the strategy $x_j : S \to X$ (in X^K) that assigns policy positions $x_j(s)$ for every signal realization $s \in S$. In the subgame associated with any pair $(x_A, x_B) \in X^2$ of candidate platforms, let V denote the set of voting subgame (pure) strategies $v : I \to \{A, B\}$ which specify a vote choice $v(s) \in \{A, B\}$ for every signal realization $s \in I$. Let Σ denote the set of strategies $\sigma : X^2 \to V$ in the complete game that specify a subgame voting strategy $\sigma(x_A, x_B) \in V$ for every pair $(x_A, x_B) \in X^2$ of candidate platforms.

Equilibrium If his peers all follow $v \in V$ in the subgame associated with (x_A, x_B), a voter's best response v^{br} maximizes his expectation of (1) (over N, z, s_A, s_B, and $(s_i)_{i=1}^N$, which, together with the voting strategy v, determine the election winner w), conditional on s_i and on whatever information he can infer about candidates' signals from observing x_A and x_B. $v^* \in V$ is a Bayes-Nash equilibrium (BNE) in the voting subgame if it is its own best response. In the broader game, (x_A^*, x_B^*, σ^*) is a perfect Bayesian equilibrium (PBE) if $\sigma^*(x_A, x_B)$ is a BNE for every subgame and $x_j^*(s_j)$ is a best response to x_{-j}^* and σ^* for every $s_j \in S$, meaning that it maximizes the expectation of (1) (over N, z, s_{-j}, $(s_i)_{i=1}^N$, and w), taking

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[4]A continuous H is convenient for ensuring a unique cutoff dividing voters who prefer one candidate or the other. Because candidates' continous policy choices depend on their signals, a discrete G is helpful for tractability. Numerical simulations in the appendix confirm that best response behavior is similar for continuous G, but do not derive equilibrium.

[5]The strict MLRP states that if $s' > s$ then $\frac{\Pr(s'|z)}{\Pr(s|z)}$ increases in z.

x_{-j}^* and σ^* into account. In equilibrium, voters infer some information about candidates' signals from their platforms. Establishing equilibrium requires specifying what voters would believe about candidate signals if candidates deviated to non-equilibrium platforms. For simplicity, consider PBE with *robust beliefs* (PBER), meaning that voters who observe a non-equilibrium platform $\hat{x}_j$ believe candidate j to have observed the signal realization $\hat{s}_j$ that minimizes the distance from $x_j(s_j)$ to $\hat{x}_j$.[6]

3 Analysis

3.1 Voting

This section focuses on the subgame associated with any pair (x_A, x_B) of candidate platforms. Below, Lemma 1 formally characterizes best response and equilibrium voting in any such subgame. The model here is more general, but the proof of Lemma 1 follows logic identical to Lemmas 1 and 2 in McMurray (2022), and so is omitted here. Proofs of subsequent results are presented in the Appendix.

With quadratic utility, a voter prefers policies as close as possible to his expectation of z, conditional on available information. By MLRP, expectations are monotonic in signals, so as Part 1 of Lemma 1 states, voting follows a threshold strategy. The indifferent voter is the one whose expectation of z lies midway between candidates' platforms.

A voter's expectation begins with his own signal s_i, but he also infers what he can from candidate platforms. For any policy $x \in X$, let $S_j(x) = \{s \in S : x_j(s) = x\}$ denote the subset of signals in S that lead candidate j to implement policy x. If voters observe candidates adopt platforms x_A and x_B, they infer from this that $s_A \in S_A(x_A)$ and $s_B \in S_B(x_B)$, and update their expectations about z accordingly. As is standard in voting models, a voter also takes into account that his own vote will change his utility only if it is *pivotal* (event P), making or breaking a tie.

Part 2 of Lemma 1 states that equilibrium in the voting subgame is unique and socially optimal, characterized by a voting threshold $t^*(\bar{x})$ that depends only on the midpoint $\bar{x} = \frac{x_A + x_B}{2}$ between candidates' platforms. This threshold increases in $\bar{x}$, and therefore in x_A and x_B. Shifting either candidate's platform to the right therefore increases support for the candidate on the left. Part 3 states that Condorcet's jury theorem holds in this environment: in a large election, the candidate closest to the optimal policy is almost sure to win.

Lemma 1. *In any voting subgame associated with* (x_A, x_B),

(1) $v^{br} \in V$ *is a best response to* $v \in V$ *only if* $v^{br}(s) = \begin{cases} \arg\min_j x_j & \text{if } s < t^{br} \\ \arg\max_j x_j & \text{if } s > t^{br} \end{cases}$ *for some voting threshold* $t^{br} \in I$ *solving* $E(z|P, s_i = t^{br}, S_A(x_A), S_B(x_B)) = \frac{x_A + x_B}{2}$.

(2) There exists a unique voting threshold $t^* : X \to I$ *such that if* $\bar{x} = \frac{x_A + x_B}{2}$ *then* $v_{t^*(\bar{x})}$ *is a subgame*

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[6]If multiple signal realizations minimize $|x_j(s), \hat{x}_j|$ then let $\hat{s}$ and $\hat{s}'$ denote the lowest and highest of these, and assume that voters believe j to have observed $\hat{s}$ or $\hat{s}'$, respectively, when they observe a platform below or above $x_j(\hat{s}) = x_j(\hat{s}')$.

equilibrium and is socially optimal in V. If $x_A \neq x_B$ then $v_{t^*(\bar{x})}$ is the only subgame equilibrium and is uniquely optimal in V. Moreover, t^* increases in $\bar{x}$, and therefore in x_A and x_B.

(3) (Jury Theorem) If $j^* = \arg\max u(x_j, z)$ then $\lim\limits_{n\to\infty} Pr_n(w = j^*) = 1$.

3.2 Platforms

Let σ^* denote the strategy that induces optimal voting $v_{t^*(\bar{x})}$ in every subgame. An equilibrium in the complete game requires this voting strategy, combined with equilibrium candidate platform strategy functions $x_j^* : S \to X$. Proposition 1 now lists necessary conditions for equilibrium and states that a PBER exists. Part 1 states that, in response to each signal, each candidate implements her expectation of the optimal policy. Part 2 states that a candidate's platform never coincides with her opponent's. Part 3 states that, in intervals between her opponent's possible platform realizations, a candidate's policy choice increases in her signal.[7]

Proposition 1. *A PBER exists. If (σ^*, x_A^*, x_B^*) is a PBER then the following hold for all $s, s' \in S$ and for $j = A, B$.*

(1) $x_j^*(s) = E_{z,s_{-j}}(z|j,s)$.

(2) $x_A^*(s) \neq x_B^*(s')$ for all $s, s' \in S$.

(3) If $x_j^*(s) < x_j^*(s')$ and $\left\{ \tilde{s} \in S : x_j^*(s) < x_{-j}^*(\tilde{s}) < x_j^*(s') \right\} = \emptyset$ then $s < s'$.

Naturally, a candidate conditions her expectation of the optimal policy on her private signal s_j. Since candidates are identical, it might seem intuitive that they should respond identically to identical signals. As in McMurray (2022), however, Part 1 of Proposition 1 states that candidates also condition on the "pivotal" event $w = j$ of winning the election. Conditioning on the event of winning may seem odd, since platform selections are made *before* votes are even cast, but if a candidate loses the election, her platform choice will not affect her utility, so she rationally restricts attention to the special circumstances in which her platform will attract more votes than her opponent's, and positions herself in a way that will be optimal in that event. If voters' strategy makes this more likely in some states of the world than others, she takes that into account. Since they condition their beliefs on opposite events (i.e., events A and B), Part 2 of Proposition 1 states that candidates *never* respond identically to the same signal. In that sense, pivotal considerations are always important in equilibrium.

Pivotal considerations substantially complicate a candidate's decision, because they reveal information that *depends* on her own behavior: winning from one policy position may reveal information that favors a different platform, but deviating to that platform may reveal new information that makes the candidate regret deviating, or makes her want to deviate further. If both candidates adopted the same degenerate

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platform strategy $x_A = x_B = E(z) = 0$, for example, they would win with equal probability and the event of winning would convey nothing about z to either candidate. Since they learn nothing to make them regret adopting the same centrist platform position, this may seem like an equilibrium, but it is not, because a candidate who deviates would then generate new information, validating the deviation.[8] In equilibrium, a candidate chooses a policy platform that will be optimal *if* she learns that she has won from that position.

Analysis is further complicated because a candidate who wins from a position left of her opponent's infers that voters have determined that z is low, while a candidate who wins from the right infers that z is high. Oddly, a policy position just left of her opponent's can therefore be too high, while a position just right of her opponent's can be too low, so that mimicking her opponent's policy locally *minimizes* expected utility. Away from x_{-j}, expected utility is concave, increasing as a candidate moves away from x_{-j} in the direction of her expectation $E(z|B, s_B)$ but then decreasing again if she passes this point. With a local minimum at x_{-j} and concave utility to the left and right of x_{-j}, expected utility is "M shaped", with local best responses both left and right of x_{-j}.

A third complication is that a candidate does not know where her opponent's platform will be: there are as many possible realizations of $x_{-j}(s)$ as there are realizations of s. Thus, expected utility is not M shaped but has "generalized M" shape, with local minima at every realization of $x_{-j}(s)$ and concave sections between, each with a local best response, as Figure 1 illustrates.[9] It is never optimal for a candidate to adopt the same policy position as her opponent, so as Part 2 of Proposition 1 states, platforms *never* coincide for *any* combination of signals. In that sense, candidates' policy positions repel one another, as can be seen in the polarized equilibria below, among others.

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[8]For example, deviating to the right and winning would suggest that the optimal policy is right of center, making the deviation beneficial.

[9]Expected utility may not be literally minimized at every $x_{-j}(s)$, but is never maximized: as her platform increases from just below to just above x_{-j}, a candidate's marginal expected utility increases discontinuously, implying that expected utility is higher just left or just right of every $x_{-j}(s)$, or both.
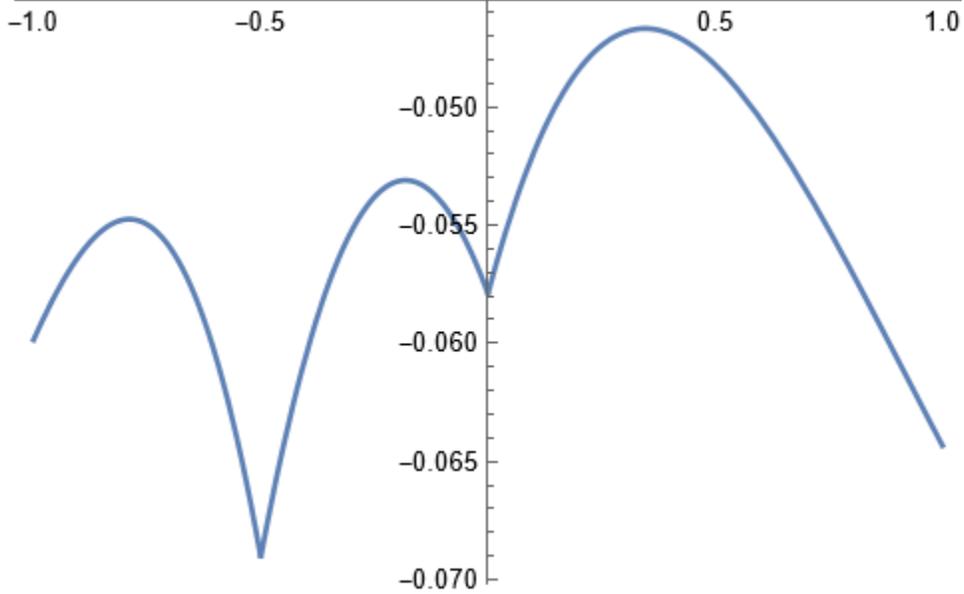
Figure 1: Expected utility $E_{w,z}\left[u\left(x_w,z\right)|s_B\right]$ as a function of $x_B \in [-1,1]$, when $x_A(s_1)= -5$ and $x_A\left(s_2\right) = 0$.

It is difficult to make general arguments regarding which of the many local best responses is a candidate's global best response. In general, $x_j^{br}\left(s\right)$ may lie in any cell of the partition created by $x_{-j} : S \to X$. However, different signal realizations shift the entire profile of expected utility for different policy platforms that a candidate might adopt, and the monotone likelihood ratio property guarantees that a candidate's expectation of z increases in her signal. Thus, best response platforms $x_j\left(s\right)$ and $x_j\left(s'\right)$ may lie in different cells of x_{-j}, but if they lie in the same cell and $s < s'$ then necessarily $x_j\left(s\right) < x_j\left(s'\right)$ as well. As Part 3 of Proposition 1 states, this feature of best responses also holds for equilibrium platform strategies.

A platform function that is locally monotonic fully reveals a candidate's private signal, both to voters and to the candidate's opponent. In equilibrium, therefore, voters vote as if they observed s_A and s_B, and every opponent position $x_{-j}\left(s_{-j}\right)$ that a candidate anticipates facing shapes her belief about the underlying truth variable z.[10] A candidate's own signal also gives some information about which opponent signals, and therefore which opponent platforms, are most likely.

3.3   Large Elections

As the number of voters grows large, the jury theorem guarantees that the candidate whose platform is truly superior almost surely wins the election. Even if candidates' signals are of much higher quality than voters', their importance in identifying the location of z therefore vanishes as the number of voters grows large. As candidate signals become less important for voters, they also become less important for the candidates themselves, who share voters' preferences. This simplifies the analysis because the probability of

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[10]In the proof of Proposition 1, voters' optimal reaction to s_A and s_B provides a useful envelope theorem-like result, ensuring that voter reactions to candidates' platform adjustments are second-order small.

a candidate winning the election in states closer to her own platform than her opponent's tends to one, while the probability of winning in other states tends to zero. Conditional on winning from platform x_j against platform x_{-j}, then, a candidate's perceived distribution of z is simply truncated at the midpoint $\bar{x} = \frac{x_j + x_{-j}}{2}$ between her own platform and her opponent's. As Proposition 2 now states, her overall posterior of z is the weighted average of these truncated conditional distributions.

Proposition 2. *If $\left(\sigma_n^*, x_{A,n}^*, x_{B,n}^*\right)$ is a sequence of PBER with limit $\left(\sigma_\infty^*, x_A^*, x_B^*\right)$ then, for all $s_j \in S$ and for $j = A, B$, $x_j^*(s_j)$ solves*

$$x_j^* = \sum_{s_{-j}:x_{-j}^*(s_{-j})<x_j^*} E\left(z|z > \bar{x}^*, s_j\right) \Pr\left(s_{-j}|s_j\right) + \sum_{s_{-j}:x_{-j}^*(s_{-j})>x_j^*} E\left(z|z < \bar{x}^*, s_j\right) \Pr\left(s_{-j}|s_j\right) \qquad (2)$$

where $\bar{x}^* = \frac{x_{-j}^*(s_{-j})+x_j^*}{2}$.

Equation 2 divides a candidate's opponent's policy positions into two groups: those that lie to the left of her own platform and those that lie to the right. Though s_j becomes unimportant for knowing the outcome of an election (given x_A, x_B, and z), it remains useful for helping a candidate anticipate which values of z are most likely, in turn determining which opponent signals s_{-j} and platforms she is most likely to face. This is reflected in the conditional probabilities $\Pr\left(s_{-j}|s_j\right)$ and conditional expectations $E\left(z|z > \bar{x}^*, s_j\right)$ in Equation 2.

A platform that solves Equation 2 is a local best response (since expected utility is locally concave) but may not be optimal globally: given the generalized M shape described in Section 3.2, a platform in a different cell of the partition created by x_{-j} might be superior. It is difficult to make general arguments about which of these many local maxima will be the global maximum; to make progress, Section 4 now considers specific F and G, for which local best responses can be determined numerically. Computing welfare for each local best response, these can then be compared to determine global best responses and Nash equilibria. This approach does not speak to generality, but does establish that pivotal considerations can produce certain behaviors of interest.

4 Numerical Analysis

4.1 Setup

In this section, let z be uniform on $X = [-1, 1]$ (that is, $f(z) = \frac{1}{2}$) and let $s_k = -1 + 2\frac{k-1}{K-1}$, so that signals are spaced evenly between $s_1 = -1$ and $s_K = 1$, with mass function $g(s|z) = \frac{1}{K}(1 + sz)$ that is simply linear in the signal s. Equilibrium is analyzed in the limit as n grows large, thus utilizing Proposition 2. Because of computational limitations, equilibria are derived only for a small number K of signal realizations.[11] By

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[11]To offer a partial sense of what might happen with more signals, the appendix also considers a continuum of signals with linear density $g(s|z) = \frac{1}{2}(1 + sz)$. That environment is not tractable enough to derive equilibrium, but that section does show that behaviors similar to the finite equilibria below have local best responses that are similar, as well.

Bayes' rule, a candidate's posterior belief is given by $f(z|s) = \frac{1}{2}(1 + sz)$, so her expectation of the optimal policy is simply $E(z|s) = \frac{1}{3}s$. Both of these are simply linear in her signal realization s. Non-strategic candidates would adopt platforms at these expectations, so this offers a benchmark with which equilibrium can be compared.

Proposition 1 states that, between her opponent's platform positions, a candidate's strategy is monotonic in her own signal. For simplicity, this section considers only strategies $x_j(s)$ that are monotonic in s everywhere.[12] To keep notation concise, let $x_{jk} = x_j(s_k)$ denote the platform choice of candidate j in response to signal s_k. For every signal realization, candidate B can position herself in any of the $K+1$ cells of the partition of X created by $x_{A1}, x_{A2}, ..., x_{AK}$.

With both candidates adopting K policy positions, there are a large number of possible policy configurations.[13] The order of these platforms can be expressed using a to denote any x_{Ak} and b to denote any x_{Bk}. With three signals, for example, the configuration $aaabbb$ denotes a configuration that is *fully polarized*, meaning that all of A's policy positions are to the left of any of B's policy positions. Configurations $aababb$ and $ababab$ are *partly polarized*, meaning that x_A tends to be left of x_B, but can be to the right. $aabbba$ is an *extremist/centrist* configuration: A overreacts to her signal by adopting a far left policy when s_A is negative and a far right policy when s_j is positive, while B underreacts to s_B, always adopting a centrist position. For different K, the sections below analylze whether configurations such as these can emerge in equilibrium.

For any K and any configuration of platforms, establishing an equilibrium requires solving a system of $2K$ equations of the form $x_{jk} = \lim_{n \to \infty} E(z|w = j, s_k; x_j, x_{-j})$ for $j = A, B$ and for $k = 1, 2, ..., K$, which are necessary (in large elections) for a local best response. Then, fixing x_{-j}, numerically evaluate expected utility for all $x \in X$ and s_k to verify that the local best response is a global best response. Section 5 provides additional details. Once an equilibrium is established, welfare is derived by averaging over s_k and integrating 1 over z.

4.2 Full Polarization

When her platform is further right than her opponent's, winning conveys to a candidate that voters have found z to be high; if it is further left, winning conveys that z is low. Not knowing whether her opponent's platform is left or right complicates a candidate's inference. If platforms are *fully polarized*, however, meaning that x_A is always left of x_B ($\max_k x_{Ak} < \min_k x_{Bk}$), then winning unambiguously conveys to A that z is low and conveys to B that z is high. Like self-fulfilling prophecies, both candidates' platforms are then appropriate for the circumstances that lead them to win.

For various small K, numerical analysis reveals a unique fully polarized equilibrium, which Figure 2 illustrates. With two signals, $(x_A^*, x_B^*) \approx ((-.59, -.27), (.27, .59))$. With three signals, $(x_A^*, x_B^*) \approx ((-.58, -.51, -.26), (.26, .51, .58))$. With four, five, or six signals, unique equilibria exist with $x_B^* \approx (.26, .47, .54, .58)$, $x_B^* \approx (.29, .46, .53, .57, .59)$, and

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[12]Intuitively, non-monotonic strategies also seem difficult to sustain in equilibrium.

[13]Since candidates in this model are ex ante identical, they can play symmetric roles in equilibrium; to avoid duplication, only configurations where the leftmost policy belongs to candidate A are reported.

$x_B^* \approx (.26, .37, .49, .53, .56, .58)$ (and x_A^* symmetric).[14] For the sake of comparison, Figure 2 also shows policy positions $(x_A^*, x_B^*) = (-.5, .5)$ for candidates with only a single platform each (equivalently, one uninformative signal $K = 1$ or no signals at all $K = 0$), derived in McMurray (2022).
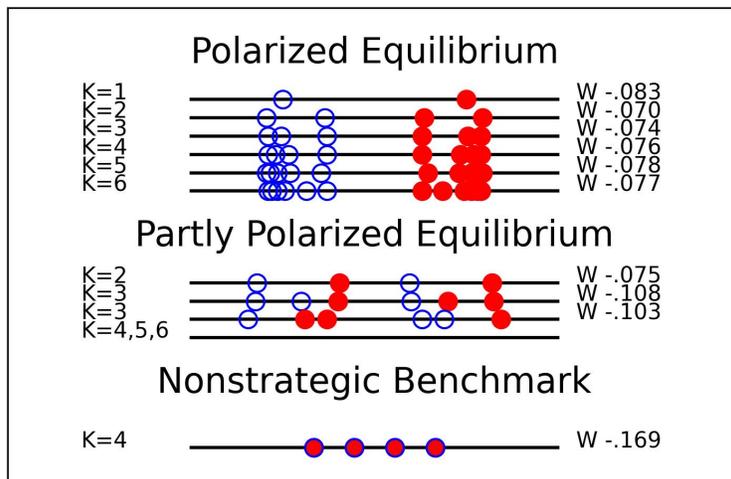

Figure 2: Polarized equilibrium policy positions and welfare, for various numbers of signal realizations.

The polarization exhibited in Figure 2 is striking: platforms range from about $\pm.3$ to about $\pm.6$; the distance between them is about 30% and 60% of the policy interval.[15] The *least* polarized platforms are almost as extreme as the *most* polarized platforms $(\pm\frac{1}{3})$ that nonstrategic candidates would adopt, who fail to take pivotal considerations into account.

Another striking feature of these fully polarized equilibria is that candidates respond *oppositely* to identical information: for the same s, $x_A(s)$ is negative and $x_B(s)$ is positive, differing by about 40% to 50% of the length of X. The correlation between $x_j(s)$ and s is less than 10^{-16} while the correlation between $x_j(s)$ and a candidate's identity j exceeds .97; in the nonstrategic benchmark, $x_A(s)$ and $x_B(s)$ are identical, so $x_j(s)$ correlates perfectly with s and not at all with j.

4.3 Partial Polarization

In the fully polarized equilibria of Section 4.2, candidates adopt extreme policy platforms and, upon winning, infer extreme information about the optimal policy. This raises the question of whether additional equilibria exist, in which candidates polarize less and, upon winning from less extreme positions, infer less extreme information about z. Say that an equilibrium is partly polarized if A's lowest and highest platform positions are lower than B's (i.e., $x_{A1} < x_{B1}$ and $x_{AK} < x_{BK}$), but the ranges of candidates' platforms overlap (i.e.,

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[14]$K > 6$ renders numerical integration and solution methods infeasible, but the appendix shows that if x_A is monotonic and always negative then B has a local best response that is monotonic and always positive.

[15]In McMurray (2017, 2022) I point out that the truth variable z underlying some policy decisions may be binary, rather than continuous. In that case, winning the election would perfectly reveal z, so regardless of their private signals, candidates would polarize to the far extremes, -1 and 1.

$\max_k x_{Ak} > \min_k x_{Bk}$), so that neither candidate can be sure whether her opponent is on her left or her right.

With two signals, there is a single configuration (*abab*) that is partly polarized, and numerical analysis finds a unique equilibrium $(x_A^*, x_B^*) \approx ((-.64, .19), (-.19, .64))$ in large elections. In this equilibrium, polarization is incomplete but still quite pronounced: platforms differ by about 20% to over 60% of the policy interval, and responses to the same signal differ by nearly 25% of the policy interval.

When $K = 3$, five platform configurations are partly polarized: *aababb*, *aabbab*, *abaabb*, *ababab*, and *abbaab*. As Figure 2 illustrates, *ababab* and *abbaab* can both occur in equilibrium: either $x_B^* = (-.40, .20, .65)$ or $x_B^* = (-.38, -.26, .69)$, with x_A^* symmetric. However, the other three configurations cannot.[16] The reason for this is that, as Proposition 1 highlights, mimicking her opponent's platform *minimizes* a candidate's utility. There is just enough space in the *ababab* and *abbaab* configurations for B to optimize between A platforms (and vice versa), and these configurations are sufficiently balanced that winning from the middle position makes it difficult to distinguish whether z is high and her opponent is low or vice versa. In contrast, in an *aababb* configuration, winning from a centrist position tells candidate B that z is high and A is low, so that a negative response can be locally optimal when $s_B = -1$ but then the global best response is positive. *abaabb* and *aabbab* are so asymmetric that B's one candidate's negative position is not even optimal locally.

With larger K, more configurations of candidates' platforms are possible, but there is less space to optimize while avoiding an opponent's platform positions, making equilibrium more difficult to sustain. With $K = 4$, for example, nineteen partly polarized configurations are possible. Not all of these are treated here, but there are seven configurations in which A and B play symmetric roles, and none of these can emerge in equilibrium.[17] $K = 5$ and $K = 6$ have even too many symmetric configurations to explore thoroughly, but first-order conditions for the fully integrated configurations *ababababab* and *abababababab* exhibit no solution, suggesting that partial polarization is unlikely to be sustainable when K is large.[18]

4.4   Extremist / Centrist Equilibrium

The asymmetry of fully and partly polarized equilibria reinforces itself: a candidate who wins from the left or right learns that z is low or high, respectively. This section highlights another equilibrium possibility, which is that an *extremist* candidate A overreacts to her signal (so that $x_A(s_A)$ is very low when s_A is low and very high when s_A is high) while the *centrist* candidate B underreacts to hers (with $x_B(s_B)$ moderate for all s_B). Winning from the far left or right gives strong evidence that z is low or high, respectively, thus reinforcing A's extremism, while winning from the center communicates little about the direction of z,

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[16]Necessary equilibrium conditions for *aababb* have a unique solution, but it is optimal only locally, not globally. For *aabbab* and *abaabb*, first-order conditions have no numerical solution.

[17]Asymmetric configurations seem unlikely to emerge in equilibrium, given the logic above that a candidate who wins from a centrist position in an asymmetric configuration has clear directional incentives that make the political center an inferior policy location.

[18]With continuous signals, marginal utility no longer increases discretely when platforms coincide, but generally speaking, local best responses still tend to avoid an opponent's platforms, as Section 5 shows, so that partly polarized equilibria remain unlikely.

leaving B content in the center.

For $K = 2$, there is a unique equilibruim $(x_A^*, x_B^*) = ((-.71, .71), (-.15, .15))$ of form *abba*, illustrated in Figure 3. In this equilibrium, candidates respond symmetrically to symmetric signals, thereby creating symmetric incentives for each other.

With $K = 3$, two almost symmetric equilibria mirror each other, with A having a heavier presence on one side and B having a heavier presence on the other: $(x_A^*, x_B^*) \approx ((-.67, -.63, .74), (-.04, .14, .28))$ and $(x_A^*, x_B^*) \approx ((-.28, -.14, .04), (-.74, .63, .67))$. If candidate A mixes in response to the most centrist signal realization, there is also an equilibrium $(x_A^*, x_B^*) \approx ((-.70, \pm.67, .70), (-.21, 0, .21))$ with perfect symmetry. Actually, there is also a mixed equilibrium for uninformative signals ($K = 0$ or $K = 1$), overlooked in McMurray (2022), in which $x_B = 0$ but A mixes between $-.67$ and $.67$.

Symmetric equilibria for $K = 4$, $K = 5$ (with mixing), and $K = 6$ are given by $(x_A^*, x_B^*) \approx ((-.71, -.69, .69, .71), (-.22, -.07, .0$ $(x_A^*, x_B^*) \approx ((-.70, -.69, \pm.67, .69, .70), (-.22, -.11, 0, .11, .22))$, and $(x_A^*, x_B^*) \approx ((-.70, -.69, -.68, .68, .69, .70), (-.22, -.14, -.05, .05, .$ All of these are illustrated in Figure 3. [19]
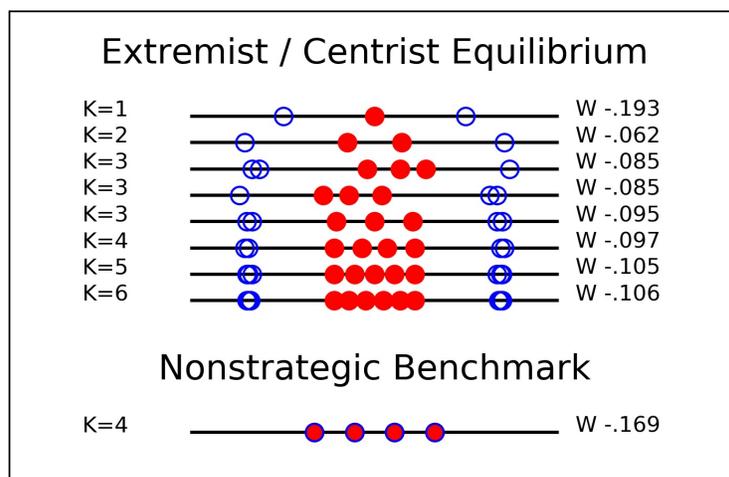

Figure 3: Extremist/centrist equilibrium policy positions and welfare, for various numbers of signal realizations.

Extremist / centrist equilibria are remarkable in that pivotal considerations create qualitative differences between candidates: one overreacts to her signal while the other reacts very little. Reactions to the same signal differ by 24% to 38% of the policy interval. Extremist positions can be twice as extreme as the nonstrategic benchmark, while centrists are less extreme.

4.5 Welfare

Since voters and candidates share a common preference in this model, it is not controversial to reinterpret expected utility as social welfare. The jury theorem guarantees that voters will identify the better of two

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[19]For higher K, computation no longer seems feasible. There is also a symmetric, mixed strategy equilibrium for $K = 0$, where A plays $-.67$ and $.67$ with equal probability and B plays 0 with certainty.

platforms, so equilibrium welfare $W = \lim_{n\to\infty} E_{w,z}\left[u\left(x_w, z\right)\right]$ in large elections can be written as the expected utility of the better of the two equilibrium platforms.

$$W = \int \sum_{S^2} g\left(s_A|z\right) g\left(s_B|z\right) \max\left\{u\left(x_A^*\left(s_A\right), z\right), u\left(x_B^*\left(s_B\right), z\right)\right\} f\left(z\right) dz \tag{3}$$

This can be computed numerically for the equilibria described above, and is displayed for each equilibrium in Figures 2 and 3. The general pattern that emerges is that fully polarized equilibria produce higher welfare than extremist / centrist equilibria, which are superior to equilibria that are only partly polarized.[20] All are much better than the non-strategic benchmark.[21] As the best equilibrium, full polarization also produces higher welfare than any non-equilibrium behavior.[22]

The jury theorem ensures that the better of the two candidates' platforms will be implemented, so the important thing for welfare is simply that candidates offer at least one platform close to the true z. The non-strategic benchmark tries to accomplish this with platforms that track candidates' private signals, in turn tracking the truth. Partially polarized equilibria improve on this by giving voters not one but two platforms in the vicinity of z, creating option value. The unfortunate possibility remains, however, that both candidates might draw signals far from the truth. Extremist / centrist equilibria insure further against this with a centrist platform that is never too far from the truth, together with an extreme platform that is likely in the right vicinity.[23] Fully polarized equilibria go further still, forgoing the second platform near z to guarantee policy options on both sides of the center.

Since voters and candidates share a common interest, the best combination of voter and candidate behavior always constitutes an equilibrium (McLennan 1998). Improving the information of voters or candidates makes more actions available to them, thus improving welfare. A policy implication of this is that voters should use any mechanisms available to improve their own information, and to recruit informed candidates. Individual voter information will not matter when the number of voters is already large, but candidate information still will: as Figures 2 and 3 illustrate, the best equilibria feature a variety of possible candidate positions; better information will help candidates position closer to the truth.

5   Conclusion

The process of identifying and implementing good policies is a joint venture between political candidates and voters. On the surface, candidates' opinions seem more central in this process than voters', but this paper has shown that strategic candidates may ignore their own policy opinions almost entirely, taking a

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[20]The one exception to this is $K = 2$, where the the extremist / centrist equilibrium is better than the fully polarized equilibrium.

[21]The one exception to this is $K = 1$, where the extremist / centrist equilibrium is even worse than the non-strategic benchmark.

[22]McLennan (1998) shows that, in common interest games such as this, welfare maximizing behavior always constitutes an equilibrium.

[23]$K = 1$ is problematic because the extremist candidate is not guided by a private signal, but polarizes randomly.

position on the left when they privately believe the far right is optimal, or vice versa. This can actually be good for society, as voters collectively have better information than either candidate, by the logic of Condorcet's (1785) jury theorem. But candidate information still matters, so voters should seek to recruit the best informed candidates they can find.

Since policy positions both affect and are affected by what candidates learn from voters, multiple equilibria can arise, including some that are inefficient and unusual. None of the ingredients in the model above were included with an aim to produce unusual behavior; it is just an inevitable consequence of rationality and two-sided private information about a shared objective. That pivotal considerations so dramatically affect voters' incentives has led many to wonder whether voters are capable of such strategic sophistication;[24] the dramatic impact of the pivotal considerations above raises the same question about candidates. Empirically, the extremist/centrist equilibria identified above are not familiar, and candidates seem unlikely to acknowledge that their opinions might be wrong, let alone that they anticipate inferring information from voters. On the other hand, the most robust consequence of the model seems to be polarization, which seems ubiquitous empirically; it may be that candidate extremism reflects confidence derived subconsciously from the expectation of voter support.[25]

If two-sided information problems become intractable but closely resemble a model with voter signals alone, excluding candidate signals may be a reasonable way forward for some purposes. Candidate information still matters in polarized equilibria, however, as emphasized above, so finding tractable ways to integrate voter and candidate information could be a useful direction for future work. Other useful directions would be to relax the assumption of pure common interest or the canonical information distributions that render large electorates infallible: if electorates have their own objectives or make mistakes, candidates may rely more on their private policy opinions than they do in the model above. As long as voters possess *any* policy information that candidates value, the pivotal considerations above will remain relevant, and will interact with candidates' policy positioning. The jury theorem embodies democracy's fundamental promise that voters make good collective decisions; to the extent that this hope is justified, voters' opinions may be more important to candidates than their own.

Appendix

Proofs

Proof of Proposition 1

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[24]In the laboratory experiments of Esponda and Vespa (2014), for example, voters have difficulty following strategic incentives even when primed to do so.

[25]Existing literature offers many explanations of polarization; for example, candidates will polarize if base supporters withdraw support from candidates who moderate. However, most such theories overlook that base supporters would be beter off encouraging moderation rather than opposing it. In contrast with existing literature, the model above assumes full rationality. As I show in McMurray (2022), an information rationale for polarization is also fairly robust to office motivation.

Proof. Part 2 of Lemma 1 states that the equilibrium voting strategy in any subgame where candidate platforms differ is socially optimal. This is true whether or not voters can infer s_A and s_B from x_A and x_B. Voters who can infer s_A and s_B still have the option of behaving as if they did not, however, so welfare is weakly higher in that case. Since voters and candidates have identical preferences, this implies that candidates, too, prefer to reveal s_j to voters (so that voters will optimally take s_j into account in formulating their voting response). If a candidate strategy calls for $x_j(s_j) = x_j(s'_j)$ for some $s_j < s'_j$, however, then robust voter beliefs imply that candidate j can reveal s_j or s'_j by deviating slightly to the left or just to the right (leaving policy utility otherwise virtually unchanged). Thus, $x_j(s_j) \neq x_j(s'_j)$ in any PBER. Since voters infer s_A and s_B and the equilibrium voting threshold t^* is chosen optimally, $\frac{\partial}{\partial t} E_{w,z}(u(x_w, z)|s_A, s_B, x_A(s_A); x_B) = 0$ for all s_A and s_B.

If candidate B observes $s_B \in S$ but cannot infer s_A and chooses platform $x_B \in X$ in response to $x_A : S \to X$ then her expected utility averages over $s_A \in S$, as follows,

$$E_{s_A, z, w}[u(x_w, z)|s_B] = E_{s_A, z}[\Sigma_{w=A,B} u(x_w, z) P(w|z, x_A(s_A); x_B)|s_B] \tag{4}$$

where win probabilities implicitly depend on candidate A's voting strategy. The first-order necessary equilibrium condition that $\frac{\partial}{\partial x_B} E_{s_A, z, w}(u(x_w, z)|s_B) = 0$ requires that $x_B = E_{s_A, z}(z|B, s_B)$, so differentiating (4) reduces as follows,

$$
\begin{aligned}
\frac{\partial}{\partial x_B} E_{s_A, z, w}(u(x_w, z)|s_B) &= E_{s_A, z}\left(\frac{\partial}{\partial x_B} u(x_B, z) P(B|z, x_A(s_A); x_B)|s_B\right) \\
&\quad + E_{s_A}\left(\frac{\partial}{\partial t} E_{w,z}(u(x_w, z)|s_A, s_B, x_A(s_A); x_B) \frac{\partial t^*(\bar{x}; x_A(s_A), x_B)}{\partial \bar{x}} \frac{\partial \bar{x}}{\partial x_B}|s_B\right) \\
&= 2[E_{s_A, z}(z|B, s_B) - x_B] P(B|s_B; x_B) \tag{5}
\end{aligned}
$$

establishing Part (1).

As I show in McMurray (2022), MLRP implies that if $x_A < x_B$, $P(B|z; t^*(\bar{x}))$ increases in z, implying in turn that $f(z|B, s_A, s_B; t^*(\bar{x}))$ first-order stochastically dominates $f(z|s_A, s_B)$, and therefore that $E(z|B, s_A, s_B; x_A, x_B) > E(z|s_A, s_B; x_A, x_B)$. Symmetrically, $x_A > x_B$ implies that $E(z|B, s_A, s_B; x_A, x_B) < E(z|s_A, s_B; x_A, x_B)$. For any signal realization $s' \in S$ and the platform $\hat{x}_A = x_A(s')$ adopted by candidate A for that signal value, therefore, $\lim_{x_B \to \hat{x}_A^-} E(z|B, s_A, s_B; \hat{x}_A, x_B) < \lim_{x_B \to \hat{x}_A^+} E(z|B, s_A, s_B; \hat{x}_A, x_B)$. This implies that $E(z|B, s_A, s_B; \hat{x}_A, x_B)$ increases discontinuously in x_B at $\hat{x}_A$. Averaging across s_A, $E_{s_A, z}(z|B, s_B; x_B)$ and therefore (5) increase discontinuously at $\hat{x}_A$ as well. Other than at this discontinuity, however, robust voter beliefs imply that expected utility is differentiable in x_B in a neighborhood of $\hat{x}_A$, implying that (5) is either negative for x_B just to the left of $\hat{x}_A$ or positive just to the right of $\hat{x}_A$, or both. Either way, $x_B = \hat{x}_A$ does not maximize B's expected utility, and so is not a best response to the strategy x_A. Thus, $x_B^{br}(s) \neq \hat{x}_A = x_A(s')$ for any $s \in S$, and therefore $x_B^*(s) \neq x_A^*(s')$, establishing Part (2). Away from this discontinuity, the monotonicity of $E_{s_A, z}(z|B, s_B; x_B)$ implies Part (3). The continuity of expected utility with respect to $x_j(s)$ for any s implies continuity in pairs (x_A, x_B) of platform strategies, the set of which is compact X is compact (and therefore $x_j : S \to X$ is compact for $j = A, B$), so a PBER exists by Brouwer's theorem.

$\square$

Proof of Proposition 2

Proof. Lemma 1 states that $x_B^{br}(s_B) = E_{s_A,z}(z|B, s_B; n)$ for every n. As n grows large, Lemma 1 implies that $P(B|z; t^*(\bar{x}), n)$ approaches 1 (where $\bar{x} = \frac{x_{A,n}(s_A) + x_B}{2}$) if z is closer to x_B than to $x_{A,n}(s_A)$ and approaches 0 otherwise. Conditional on s_A such that $x_A(s_A) = \lim_{n\to\infty} x_{A,n}(s_A) < x_B$, therefore,

$$E_z[z|B, s_A, s_B; t^*(\bar{x}), n] = \int_{-1}^{1} z \frac{f(z|s_A, s_B) P(B|z; t^*(\bar{x}), n)}{\int_{-1}^{1} f(z|s_A, s_B) P(B|z; t^*(\bar{x}), n) dz}$$

approaches the following

$$\lim_{n\to\infty} E_z[z|B, s_A, s_B; t^*(\bar{x}), n] = \int_{\bar{x}}^{1} z \frac{f(z|s_A, s_B)}{\int_{\bar{x}}^{1} f(z|s_A, s_B) dz}$$

$$= E_z[z|z < \bar{x}, s_A, s_B; x_A(s_A), x_B]$$

(where now, abusing notation, $\bar{x} = \frac{x_A(s_A) + x_B}{2}$); if $x_A(s_A) > x_B$ then it approaches $\lim_{n\to\infty} E_z[z|B, s_A, s_B; t^*(\bar{x}; s_A, s_B), n] = E_z[z|z > \bar{x}, s_A, s_B; x_A(s_A), x_B]$.

$E(z|B, s_B; n) = E_{s_A}[E(z|B, s_A, s_B; t^*(\bar{x}), n)]$ averages across all s_A, so it approaches the weighted average of these limits.

$$\lim_{n\to\infty} E(z|B, s_B; t^*(\bar{x}), n) = \sum_{s_A : x_{A,n}(s_A) < x_B} E(z|z > \bar{x}) P(s_A|s_B) + \sum_{s_A : x_{A,n}(s_A) > x_B} E(z|z < \bar{x}) P(s_A|s_B)$$

Candidate A's expectation is analogous. The solution $\left(x_{A,n}^{br}, x_{B,n}^{br}\right)$ to $x_A = E_{s_B,z}[z|B, s_A; t^*(\bar{x}), n]$ and $x_B = E_{s_A,z}[z|B, s_B; t^*(\bar{x}), n]$ therefore approaches the solution $\left(x_A^{br}, x_B^{br}\right)$ to (2), as claimed.

$\square$

Numerical Analysis

Setup

Two signals The simplest version of this setup is $K = 2$. In that case, $S = \{s_1, s_2\} = \{-1, 1\}$ and $g(s|z) = \frac{1}{2}(1 + sz)$. As a benchmark, nonstrategic candidates who fail to take pivotal considerations into account implement $E(z|s) = \frac{1}{3}s$. Welfare (3) then reduces to $W = \int_{-1}^{1} \sum_{s_A=-1}^{1} \sum_{s_B=-1}^{1} \max\left\{u\left(\frac{1}{3}s_A, z\right), u\left(\frac{1}{3}s_B, z\right)\right\} f(z) g(s_A|z) g(s_B|z) dz \approx -.169$ for large n.

Platforms $x_{A1} < x_{A2}$ partition X into three segments, and candidate B can respond with a policy in $[-1, x_{A1}]$, $[x_{A1}, x_{A2}]$, or $[x_{A2}, 1]$. In a large election, these options yield the following expectations.[26] Candidate A forms expectations symmetrically.

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[26]For simplicity, this paper considers only monotonic platform strategies; otherwise, the expectation $\lim_{n\to\infty} E(z|B, s_B; x_{A2} < x_B < x_{A1})$ would also be relevant.

$$\lim_{n\to\infty} E\left(z|B, s_B; x_B < x_{A1} < x_{A2}\right) = \frac{\int_{-1}^{\frac{x_{A1}+x_B}{2}} zg\left(s_1|z\right) f\left(z|s_B\right) dz + \int_{-1}^{\frac{x_{A2}+x_B}{2}} zg\left(s_2|z\right) f\left(z|s_B\right) dz}{\int_{-1}^{\frac{x_{A1}+x_B}{2}} g\left(s_1|z\right) f\left(z|s_B\right) dz + \int_{-1}^{\frac{x_{A2}+x_B}{2}} g\left(s_2|z\right) f\left(z|s_B\right) dz} \tag{6}$$

$$\lim_{n\to\infty} E\left(z|B, s_B; x_{A1} < x_B < x_{A2}\right) = \frac{\int_{\frac{x_{A1}+x_B}{2}}^{1} zg\left(s_1|z\right) f\left(z|s_B\right) dz + \int_{-1}^{\frac{x_{A2}+x_B}{2}} zg\left(s_2|z\right) f\left(z|s_B\right) dz}{\int_{\frac{x_{A1}+x_B}{2}}^{1} g\left(s_1|z\right) f\left(z|s_B\right) dz + \int_{-1}^{\frac{x_{A2}+x_B}{2}} g\left(s_2|z\right) f\left(z|s_B\right) dz} \tag{7}$$

$$\lim_{n\to\infty} E\left(z|B, s_B; x_{A1} < x_{A2} < x_B\right) = \frac{\int_{\frac{x_{A1}+x_B}{2}}^{1} zg\left(s_1|z\right) f\left(z|s_B\right) dz + \int_{\frac{x_{A2}+x_B}{2}}^{1} zg\left(s_2|z\right) f\left(z|s_B\right) dz}{\int_{\frac{x_{A1}+x_B}{2}}^{1} g\left(s_1|z\right) f\left(z|s_B\right) dz + \int_{\frac{x_{A2}+x_B}{2}}^{1} g\left(s_2|z\right) f\left(z|s_B\right) dz} \tag{8}$$

Equation (6) takes this form because candidate B wins with a platform $x_B \in [-1, x_{A1}]$ if A observes s_1 and $z < \frac{x_{A1}+x_B}{2}$ or if A observes s_2 and $z < \frac{x_{A2}+x_B}{2}$. (7) holds because $x_B \in (x_{A1}, x_{A2})$ wins if $s_A = s_1$ and $z > \frac{x_{A1}+x_B}{2}$ or if $s_2 = A$ and $z < \frac{x_{A2}+x_B}{2}$. (8) holds because $x_B \in [x_{A2}, 1]$ wins if $s_A = s_1$ and $z > \frac{x_{A1}+x_B}{2}$ or if $s_A = s_2$ and $z > \frac{x_{A2}+x_B}{2}$.

Three or more signals If $K = 3$ then $S = \{-1, 0, 1\}$ and $g\left(s|z\right) = \frac{1}{3}\left(1 + sz\right)$. There are now four expectations analogous to (6) through (8), giving the limits of $E\left(z|B, s_B; x_B < x_{A1} < x_{A2} < x_{A3}\right)$, $E\left(z|B, s_B; x_{A1} < x_B < x_{A2} < x_{A3}\right)$, $E\left(z|B, s_B; x_{A1} < x_{A2} < x_B < x_{A3}\right)$, and $E\left(z|B, s_B; x_{A1} < x_{A2} < x_{A3} < x_B\right)$, which each average over three realizations of s_A, not just two. As before, symmetric expressions describe the expectation of candidate A. If $K = 4$ then $S = \left\{-1, -\frac{1}{3}, \frac{1}{3}, 1\right\}$ and $g\left(s|z\right) = \frac{1}{4}\left(1 + sz\right)$, and there are five expectations analogous to (6) through (8). Higher K are analogous: in general, there are $K + 1$ first-order necessary conditions for a local best response, corresponding to the $K + 1$ cells $[-1, x_A\left(s_1\right)], [x_A\left(s_1\right), x_A\left(s_2\right)], ..., [x_A\left(s_K\right), 1]$ of the partition created by candidate A's policy function strategy $x_A \in X^K$, in which B could adopt a policy platform.

Full Polarization

Two signals When $K = 2$, full polarization requires that $\max\{x_{A1}, x_{A2}\} < \min\{x_{B1}, x_{B2}\}$, and therefore (by Part 3 of Proposition 2) that $x_{A1} < x_{A2} < x_{B1} < x_{B2}$ (i.e., *aabb*, in the terminology introduced above). If such an equilibrium exists, it can be identified using (8) to solve the system of first-order necessary conditions,

$$\lim_{n\to\infty} E\left(z|B, s_B = s_1; x_{A1} < x_{A2} < x_{B1}\right) = x_{B1}$$
$$\lim_{n\to\infty} E\left(z|B, s_B = s_2; x_{A1} < x_{A2} < x_{B2}\right) = x_{B2}$$

along with analogous equations for candidate A. Numerically, this system has a unique solution $(x_A^*, x_B^*) \approx ((-.59, -.27), (.27, .59))$. Conditional on s_B, expected utility $W\left(x_A, x_{Bk}|s_k\right) = \lim_{n\to\infty} E_{w,z}\left[u\left(x_A, x_B\right)|s_B = s_k; n\right]$ from following this strategy is as follows.

$$\begin{aligned} W\left(x_A, x_{Bk}|s_k\right) = & \int_{-1}^{\frac{x_{A1}+x_{B1}}{2}} \left[P\left(s_1|z\right) u\left(x_{A1}, z\right) + P\left(s_2|z\right) u\left(x_{A2}, z\right)\right] f\left(z|s_k\right) dz \\ & + \int_{\frac{x_{A1}+x_{B1}}{2}}^{\frac{x_{A2}+x_{B1}}{2}} \left[P\left(s_1|z\right) u\left(x_{Bk}, z\right) + P\left(s_2|z\right) u\left(x_{A2}, z\right)\right] f\left(z|s_k\right) dz \\ & + \int_{\frac{x_{A2}+x_{B1}}{2}}^{1} u\left(x_{Bk}, z\right) f\left(z|s_k\right) dz \end{aligned} \tag{9}$$

Integrating numerically yields $W(x_A^*, x_{B1}^*|s_1) \approx -.161$ and $W(x_A^*, x_{B1}^*|s_2) \approx -.656$.

So far, this only establishes $x_{B1} \approx .27$ as a local best response in $(-.27, 1]$. Maintaining that $x_A \approx (-.59, -.27)$ and using appropriately modified first-order conditions, additional local best responses $x_{B1} \approx -.84$ and $x_{B1} \approx -.41$ can be derived in $[-1, -.59)$ and $(-.59, -.27)$, respectively. However, these produce lower welfare $W(x_A^*, x_{B1} \approx -.84|s_1) \approx -.173$ and $W(x_A^*, x_{B1} \approx -.41|s_1) \approx -.176$, establishing $x_{B1} \approx .27$ as the global best response to x_A^* when $s_B = s_1$. Similarly, $x_{B2} \approx -.73$ is a local best response in $[-1, -.59)$ but produces lower welfare $W(x_A, x_{B2} \approx -.73|s_2) \approx -.176$, and there is no local best response in $(-.59, -.27)$, establishing $x_{B2} \approx .59$ as the global best response when $s_B = s_2$. This establishes $x_B \approx (.27, .59)$ as the global best response in X^K to $x_A \approx (-.59, -.27)$ and, by symmetry, establishes $(x_A^*, x_B^*) \approx ((-.59, -.27), (.27, .59))$ as an equilibrium.

Three or more signals If $K = 3$ then an equilibrium configuration $aaabbb$ must satisfy $\lim_{n \to \infty} E(z|B, s_B = s_k; x_{A1} < x_{A2} < x_{A3} < x_{B1} < x_{B,k}$ for $k = 1, 2, 3$. This yields a unique numerical solution $(x_A^*, x_B^*) \approx ((-.58, -.51, -.26), (.26, .51, .58))$. Local best responses in $[-1, -.58]$, $[-.58, -.51]$, and $[-.51, -.26]$ do not produce higher expected utility for any s_k, establishes x_B as a global best response to x_A; symmetric reasoning ensures that x_A is a global best response to x_B, establishing this as an equilibrium.

Higher K can be handled similarly: first-order conditions for $K = 4$ through $K = 6$ (configurations $aaaabbbb$, $aaaaabbbbb$, and $aaaaaabbbbbb$) have unique solutions $x_B^* \approx (.26, .47, .54, .58)$, $x_B^* \approx (.29, .46, .53, .57, .59)$, and $x_B^* \approx (.26, .37, .49, .53, .56, .58)$ (with x_A^* symmetric).[27] Fixing x_A and identifying local best responses for x_B in other cells of the partition created by x_A does not improve expected utility for any s_k, establishing these as global best responses and, by symmetry, as equilibria.

Partial Polarization

Two signals If $K = 2$, an equilibrium with $x_{A1} < x_{B1} < x_{A2} < x_{B2}$ (i.e., $abab$) can be derived by using (7) and (8) to equate $\lim_{n \to \infty} E(z|B, s_1; x_{A1} < x_{B1} < x_{A2}) = x_{B1}$ and $\lim_{n \to \infty} E(z|B, s_1; x_{A1} < x_{A2} < x_{B2}) = x_{B2}$, along with analogous equations for candidate A. This yields a unique numerical solution $(x_A^*, x_B^*) \approx ((-.64, .19), (-.19, .64))$. Other local best responses can be shown to be inferior, establishing this as an equilibrium.

Three or more signals When $K = 3$, five monotone platform configurations are partly polarized: $aababb$, $aabbab$, $abaabb$, $ababab$, and $abbaab$.[28] For $ababab$, limiting first-order conditions for equilibria of the form $ababab$ are $\lim_{n \to \infty} E(z|A, s_1; x_{A1} < x_{B1} < x_{B2} < x_{B3}) = x_{A1}$, $\lim_{n \to \infty} E(z|A, s_2; x_{B1} < x_{A2} < x_{B2} < x_{B3}) = x_{A2}$, and $\lim_{n \to \infty} E(z|A, s_3; x_{B1} < x_{B2} < x_{A3} < x_{B3}) = x_{A3}$ and analogous conditions for candidate B can be formulated as in (6), (7), and (8), with a unique solution $x_B^* = (-.40, .20, .65)$ (and $x_A^* = -x_B^*$). Other local best responses in $[-1, -.65]$, $[-.65, -.20]$, $[-.20, .40]$, and $[.40, 1]$ can be shown to produce lower expected utility for any s_k, establishing x_B^* as a global best response and, by symmetry, establishing (x_A^*, x_B^*) an equilibrium.

Analogous first-order conditions for configuration $abbaab$ have unique solution $x_B^* = (-.38, -.26, .69)$

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[27]Numerical integration and solution methods become infeasible for $K > 6$.
[28]Non-monotonic configuration such as $x_{A2} < x_{A3} < x_{B1} < x_{A1} < x_{B2} < x_{B3}$, not treated here, seem unlikely to qualify as global best responses.

(with $x_A^* = -x_B^*$), and other local best responses can be shown to produce lower expected utility, establishing these as equilibria. Remaining configurations cannot emerge in equilibrium: the first-order conditions for *aababb* have a unique solution, but it is optimal only locally, not globally.[29] For *aabbab* and *abaabb*, first-order conditions have no numerical solution.

Extremist / Centrist Equilibrium

Two signals For $K = 2$, an extremist/centrist equilibrium of form *abba* can be identified using (7) and expressions analogous to (6) and (8) to solve $\lim_{n \to \infty} E(z|A, s_1; x_{A1} < x_{B1} < x_{B2}) = x_{A1}$, $\lim_{n \to \infty} E(z|A, s_2; x_{B1} < x_{B2} < x_A) = x_{A2}$, and $\lim_{n \to \infty} E(z|B, s_k; x_{A1} < x_{Bk} < x_{A2}) = x_{Bk}$ for $k = 1, 2$. This yields a unique solution $(x_A^*, x_B^*) \approx ((-.71, .71), (-.15, .15))$. Other local best responses produces lower expected utility, establishing this as an equilibrium.

Three or more signals With $K = 3$, there are two extremist/centrist configurations: *aabbba* and *abbbaa*. An equilibrium of the first type can be identified by first-order conditions $\lim_{n \to \infty} E(z|B, s_k; x_{A1} < x_{A2} < x_{Bk} < x_{A3}) = x_{Bk}$ for $k = 1, 2, 3$, $\lim_{n \to \infty} E(z|A, s_k; x_{Ak} < x_{B1} < x_{B2} < x_{B3}) = x_{Ak}$ for $k = 1, 2$, and $\lim_{n \to \infty} E(z|A, s_k; x_{B1} < x_{B2} < x_{B3} < x_{A3}) = x_{A3}$. These yield a unique numerical solution $(x_A^*, x_B^*) \approx ((-.67, -.63, .74), (-.04, .14, .28))$, and other local best responses for either player produce lower expected utility, establishing this as an equilibrium. Symmetrically, $(x_A^*, x_B^*) \approx ((-.28, -.14, .04), (-.74, .63, .67))$ is an equilibrium, as well.

If candidate A mixes in response to the most centrist signal realization, there is a third equilibrium $(x_A^*, x_B^*) \approx ((-.70, \pm.67, .70), (-.21, 0, .21))$ with perfect symmetry. This can be identified by solving the same first-order conditions as above, except solving *both* $\lim_{n \to \infty} E(z|A, s_2; x_{A2} < x_{B1} < x_{B2} < x_{B3}) = x_{A2}$ and $\lim_{n \to \infty} E(z|A, s_2; x_{B1} < x_{B2} < x_{B3} < x_{A2}) = x_{A2}$, and reformulating $\lim_{n \to \infty} E(z|B, s_k; x_{A1} < x_{A2} < x_{Bk} < x_{A2} < x_{A3})$ to take into account that x_{A2} may be in either of two locations. As before, other local best responses (for either candidate) can be shown to produce lower expected utility, establishing this as an equilibrium. By similar logic, there is also a symmetric, mixed strategy equilibrium for $K = 1$, overlooked in McMurray (2022), with $(x_A^*, x_B^*) \approx (\pm.67, 0)$.

Repeating this methodology, $(x_A^*, x_B^*) \approx ((-.71, -.69, .69, .71), (-.22, -.07, .07, .22))$, $(x_A^*, x_B^*) \approx ((-.70, -.69, \pm.67, .69, .70), (-.2$ and $(x_A^*, x_B^*) \approx ((-.70, -.69, -.68, .68, .69, .70), (-.22, -.14, -.05, .05, .14, .22))$ can be shown to be symmetric equilibria for $K = 4$, $K = 5$ (with mixing), and $K = 6$. For higher K, computation no longer seems feasible.

Continuous Signals

As K grows large, $G(s|z)$ approaches the cdf of the density function $g(s|z) = \frac{1}{2}(1 + sz)$, defined on $S = [-1, 1]$. With continuous signals, however, equilibrium must satisfy a continuum of first-order conditions, which is beyond the scope of this paper. To make some progress, this section analyzes best responses, showing that behavior similar to Nash equilibria in the finite model generates similar best response behavior even with continuous signals.

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[29]If $x_A = -x_B, x_B \approx (-.06, .54, .60)$ solves the first-order conditions but $x_{B1}^{br} \approx .25$.

Full polarization For continuous behavior similar to a fully polarized equilibrium from the finite model, consider first a strategy $x_A(s) = \frac{1}{2}(-1+s)$ for candidate A that is linear in s, making x_A uniform on $[-1,0]$. If B responds with a policy above zero then she wins with probability one if z exceeds $\bar{x} = \frac{x_A(s_A)+x_B}{2} = \frac{1}{4}(s_A + 2x_B - 1)$ and with probability zero otherwise, so her expectation (conditional on winning) is

$$E(z|B, s_B = s; x_B) = \frac{\int_{-1}^{1}\int_{-1}^{1} zf(z)g(s_A|z)g(s|z)\Pr(B|s_A, z; x_B)\,dz\,ds_A}{\int_{-1}^{1}\int_{-1}^{1} f(z)g(s_A|z)g(s|z)\Pr(B|s_A, z; x_B)\,dz\,ds_A} \tag{10}$$

and approaches

$$\lim_{n\to\infty} E(z|B, s; x_B) = \frac{\int_{-1}^{1}\int_{\frac{1}{4}(s_A+2x_B-1)}^{1} zf(z)g(s_A|z)g(s|z)\,dz\,ds_A}{\int_{-1}^{1}\int_{\frac{1}{4}(s_A+2x_B-1)}^{1} f(z)g(s_A|z)g(s|z)\,dz\,ds_A}$$

as n grows large. The numerical solution $x_B^{br}(s_B)$ to the first-order best response condition $\lim_{n\to\infty} E(z|B, s_A, s_B; x_B^{br}) = x_B^{br}$ ranges with s_B from about .20 to .56. The range of $x_B^{br}(s_B)$ is smaller than the range of $x_A(s_A)$, but if x_A instead ranges only over $[-.6, -.3]$ then x_B^{br} ranges from about .3 to about .6 as well. This does not constitute an equilibrium (since x_A is linear in s_A but x_B is nonlinear in s_B) but is in a range similar to the fully polarized equilibria for finite K.

Partial polarization For continuous behavior similar to a partly polarized equilibrium from a finite model, consider a strategy $x_A(s) = s$ that ranges linearly from $x_A(-1) = -1$ to $x_A(1) = 1$, so that candidate A may be liberal, moderate, or conservative. If candidate B responds by adopting platform x_B after observing s_B then she will win the election if it turns out that s_A and x_A were low while z was high, or that s_A and x_A were high while z was low. Upon winning, her updated expectation (10) therefore approaches the following.

$$E(z|B, s_B; x_B) \to \frac{\int_{-1}^{x_B}\int_{\frac{s_A+x_B}{2}}^{1} zf(z)g(s_A|z)g(s_B|z)\,ds_A\,dz + \int_{x_B}^{1}\int_{-1}^{\frac{x_B+s_A}{2}} zf(z)g(s_A|z)g(s_B|z)\,ds_A\,dz}{\int_{-1}^{x_B}\int_{\frac{s_A+x_B}{2}}^{1} f(z)g(s_A|z)g(s_B|z)\,ds_A\,dz + \int_{x_B}^{1}\int_{-1}^{\frac{x_B+s_A}{2}} f(z)g(s_A|z)g(s_B|z)\,ds_A\,dz} \tag{11}$$

Solving $E(z|B, s_B; x_B) = x_B$ numerically yields values ranging from $x_B^{br}(-1) \approx -.4$ to $x_B^{br}(1) \approx .4$. That the best response to a strategy with full support has far less than full support suggests, in line with the conjecture based on finite s, that partial polarization may be difficult to sustain in equilibrium for large (or infinite) K.

Extremist/centrist strategies If candidate A adopts a "centrist" linear strategy $x_A(s_A) = .4s_A$ that ranges only from $-.4$ to $.4$ and B observes a perfectly centrist private signal $s_B = 0$ then, by symmetry, a centrist platform $x_B = 0$ produces a centrist expectation $E(z|B, s_B) = 0$ conditional on winning. However, $x_B = 0$ is not B's best response, because a platform above .4 provides higher utility. In that case, $x_A(s_A)$ is always to the left, so (10) approaches

$$\lim_{n\to\infty} E(z|B, s_B; x_B) = \frac{\int_{-1}^{1}\int_{\frac{.4s_A+x_B}{2}}^{1} zf(z)g(s_A|z)g(s_B|z)\,dz\,ds_A}{\int_{-1}^{1}\int_{\frac{.4s_A+x_B}{2}}^{1} zf(z)g(s_A|z)g(s_B|z)\,dz\,ds_A}$$

as n grows large, and the numerical solution to $E(z|B, s_B; x_B) = x_B$ is $x_B^{br}(0) \approx .67$. $x_B^{br}(s_B)$ is higher for higher s_B, up to $x_B^{br}(1) \approx .70$. $x_B^{br}(0) \approx -.67$ is equally a best response when s_B is centrist, and the best responses to negative signals range down to $x_B^{br}(-1) \approx -.70$. Thus, extreme policies on either side of the spectrum best response to a centrist strategy.

If x_A lies in $[-.70, -.68]$ for negative signals and $[.68, .70]$ for positive signals (as it does in the centrist/extremist equilibrium for finite K) then, from a centrist position, B will win if s_A and x_A are low but z is high, or if s_A and x_A are high but z is low. B's best response can then be derived by equating (11) to x_B. Numerical solutions range from $x_B^{br}(-1) \approx -.24$ to $x_B^{br}(1) \approx .24$. This range is similar to the range of platforms adopted in the centrist/extremist equilibrium for finite K. Thus, just as the best response to centrist behavior is extreme, the best response to extreme behavior is centrist.

Statements and Declarations

I have no conflicts of interest related to the content of this paper.

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