“Bid Shopping” in Procurement Auctions with Subcontracting

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Abstract

We analyze the equilibrium effects of “bid shopping” – a contractor soliciting a subcontractor bid for part of a project prior to a procurement auction, then showing that bid to a competing subcontractor in an attempt to secure a lower price. Such conduct is widely criticized as unethical by professional organizations, and has been the target of legislation at both the federal and state level, but is widespread in procurement auctions in many places. Our baseline model suggests that bid shopping that brings in new subcontractors after the auction – rather than diverting some existing subcontractors to post-auction competition – always increases social surplus, benefitting the procurer at the expense of the existing subcontractors. Bid shopping that causes some subcontractors to wait to bid until after the auction tends to decease total surplus when subcontractors' bid preparation costs are low, but may increase total surplus when bid preparation costs are high.

Keywords: procurement, subcontracting
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1 Introduction

A large volume of construction, engineering and industrial projects, such as buildings, utilities, roadways and bridges, is acquired by the public sector through procurement. For example, in 2018, U.S. federal, state and local government spending on construction alone was over 300 billion dollars.[1] Contracts for procurement are usually put up for bid in an auction. Indeed, to protect the public interest against overcharging, and to prevent abuses such as fraud or favoritism, governmental agencies are typically required to solicit bids and award the prime contract to the lowest bidder. Procurement is also common in the private sector, for example for the construction of factories and office buildings, and similar concerns often motivate businesses to solicit competing bids for procurement.

Prime contractors – general contracting firms who bid for large government projects – are often unable to, or not licensed to, perform all the work on a project themselves, and therefore solicit the services of subcontractors. In construction, for example, work on concrete, steel, roofing, drywall, mechanical, electrical and plumbing is usually contracted out. On large projects, 40 to 100 subcontractors could be involved in construction (Stipanowich, 1998, p. 474), typically performing between 60% to 90% of the work (Hinze and Tracey, 1994, p. 274 and GAO, 2015, p. 1). In an effort to secure the lowest possible cost and thereby increase the likelihood they'll be awarded the project, prime contractors typically put the subcontracted portion of the work up for bid, soliciting competing bids from subcontractors ("sub-bids") before the main procurement auction ("prime auction") occurs.

In the U.S. and many other jurisdictions, however, an asymmetry exists in the legal treatment of these pre-auction subcontracting arrangements. Since the prime contractor relies on the subcontractor's proposal in preparing his bid, the subcontractor's proposal is viewed as a binding commitment, and the subcontractor can be held to honor it (Drennan v. Star Paving Co. (1958)), even if it contained a mistake. On the other hand, U.S. courts have consistently held that since the prime contractor cannot accept a subcontractor's proposal "unless and until it knows that its offer has been accepted by the owner" – the government or private entity holding the prime auction (Gregory and Travers, pp. 30-31) – the winning prime contractor is *not* bound to use the subcontractors it relied on in preparing its bid. Thus, a winning prime contractor can try to lower its costs by disclosing existing subcontractor bids to competing subcontractors to see if they can do the work at a lower price, a practice called "bid shopping" – or demanding subcontractors lower their sub-bids under the threat of replacing them, a practice called "bid chopping" or "bid chiseling."

Bid shopping could potentially occur either before or after the auction. In fact, in response to the threat of pre-auction bid shopping, many subcontractors do not submit their sub-bids until very close to the deadline for bids in the prime auction.[2] There are no such obvious "countermeasures" available for post-auction bid shopping, however – attempts to disallow it explicitly by contract have failed legally.[3] A prime contractor could commit voluntarily to honor his lowest pre-auction

[1]Federal government spending on construction was $21 Bn in 2018, and state and local government spending was $285 Bn. US Census Bureau, Construction Spending, Nov. 1, 2019.

[2]Korman et al., 1992, p. 26 contains a vivid description of sub-bids arriving in the minutes before the prime auction deadline, which we excerpt in the next section.

[3]For example, West Construction v Florida Blacktop (Fla. 4th Dist. Ct. App., 2012) concerned a subcontractor whose sub-bid contained the provision, "In the event the "buyer" in any way uses Florida Blacktop, Inc.'s bid ...

sub-bids, but might hesitate because as we discuss below, there are legitimate business reason for not always working with the lowest bidder.

While there is some uncertainty over the exact prevalence of bid shopping and other related practices, we'll show evidence in the next section that they are widespread. Survey evidence suggests bid shopping is commonplace.[4] Litigation about bid shopping is common (Goldberg, 2011, p. 552). Attempts to introduce legislation to curb bid shopping date back to 1932 (Schueller, 1960, p. 504); since 2000, every Congress but one has had legislation before it to effectively end bid shopping in federal procurement, only to see it die in chambers (more detail below). Twenty-nine U.S. states have passed laws meant to halt bid shopping, and other countermeasures (such as the use of "bid depositories," discussed below) have been tried as well. And professional organizations involved in construction in the U.S. strongly condemn bid shopping, going so far as to call the practice unethical.[5]

But while there have been efforts to stop bid shopping through contract clauses, bid depositories, legislation, and professional codes of conduct, no one to our knowledge has properly analyzed the actual effects of bid shopping in equilibrium. These effects are not obvious. A report by the U.S. Senate in 1955 claimed that "As long as subcontractors will not submit their final price prior to the award of the prime contract because of bid shopping after the award, the Government cannot get the full benefit of the low competitive price" (U.S. Senate Committee on the Judiciary, 1955, p. 8). But if prime contractors anticipate that post-award bid shopping will lower their costs, in equilibrium they will base their prime bid on that lower expected cost; if bid shopping leads to lower actual costs, at least some of those savings would likely be passed on to the procurer. On the other hand, fear of bid shopping, and anticipation of the need to lower their bid ex post, could lead subcontractors to inflate their pre-auction sub-bids, and could lower their willingness to participate at all.[6] The net effect of bid shopping on social surplus, on the likelihood of successful procurement, and on the price paid by the procurer are not obvious at a glance.

This, then, is the goal of our paper: to understand the equilibrium effects of bid shopping. We introduce a tractable theoretical model of pre-auction subcontracting and bidding. Our model

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such action(s) shall in all instances constitute acceptance of Florida Blacktop, Inc.'s bid and shall create a binding contract between the parties consistent with the bid documents." The prime contractor did indeed integrate Florida's bid into its own proposal, won the prime contract, then showed Floriday's bid to a competing subcontractor and subcontracted with them at a lower price; the court ruled Florida had no recourse, because unilateral attempts to bind the general contractor are invalid unless the general agrees in advance (Smith, Currie and Hancock LLP, 2013).

[4] Gregory and Travers (2010, p.29) report on a 2008 survey of U.S. and Canadian construction contractors, which found that 80% of respondents knew of others who has engaged in bid shopping or bid pedding, with "an amazing 32%" admitting they had done so themselves; Thurnell and Lee (2009) found that in a 2009 survey of Auckland subcontractors, "all participants indicated that bid shopping in the electirical and mechnical services trades occurred in at least 30% of all tenders for which quotations were submitted."

[5] For example, 1995 guidelines issued jointly by the Association of General Contractors of America, the American Subcontractors Association and the Associated Specialty Contractors refer to bid shopping and bid peddling as "abhorrent business practices that threaten the integrity of the competitive bidding system" and goes on to say that "The bid amount of one competitor should not be divulged to another before the award of the subcontract or order, nor should it be used by the contractor to secure a lower proposal from another bidder on that project (bid shopping)," later calling the practice "unethical." Similarly, the Code of Ethics of the American Society of Professional Estimators prohibits professional estimators from participating in bid shopping, stating that "this practice is unethical, unfair and is in direct violation of this Code of Ethics."

[6] The 1955 Senate report found that 75% of electrical contractors surveyed avoid sub-bidding on Federal construction jobs, of which "93% gave the prevalence of bid shopping as their reason" (U.S. Senate Committee on the Judiciary, 1955, p. 7).

considers a procurement auction for a project with a piece that must be subcontracted. A fixed number of prime contractors are eligible to bid in the prime auction, and each prime contractor has its own set of subcontractors it can subcontract with. For tractability, prime contractors have identical costs to complete the non-subcontracted part of the project; subcontractors have independent private costs to complete the subcontracted piece, as well as a common cost to prepare a bid.

We use this model to study the equilibrium effect of permitting post-auction bid shopping, i.e., of switching from a regime where no prime contractor would bid-shop to a regime where every prime contractor, should they win, would approach one or more additional subcontractors. We find that in the "simple" case where the winning prime contractor will approach a single new subcontractor after the auction,

- bid shopping that involves new subcontractors (and therefore does not affect the number of subcontractors bidding before the auction) does not change the likelihood the project is successfully procured, and increases total surplus

- bid shopping that diverts one or more existing subcontractors into post-auction bidding can have either a positive or negative effect on total surplus – the effect tends to be positive when bid preparation costs are high, negative when bid preparation costs are low

In addition, if the winning prime contractor will approach *multiple* new subcontractors after the auction, bid shopping *increases* the likelihood the project is procured (fixing the number of pre-auction subcontractors), and increases total surplus further. Thus, our headline results seem to largely argue in favor of bid shopping, especially when bid preparation costs are substantial. However, these results do come with several important caveats:

- Not everyone benefits from bid shopping. In the simple case without diversion, the procurer benefits, through lower prices; existing subcontractors lose out; and the effect on prime contractors' profits can be positive or negative, depending on the details of the environment.

- With bid shopping, there are multiple symmetric equilibria; our results are based on the "best" one, the only symmetric equilibrium surviving a common refinement. A less optimistic equilibrium selection could reverse some of our positive welfare results.

- If bid shopping introduced new costs borne by either the winning prime contractor or the incumbent subcontractor, this could reduce or even reverse the surplus gain.

In addition, bid shopping may raise concerns that are outside of our model. Some fear that post-award competition leads subcontractors to skimp on quality in an attempt to offer a price low enough to win the work.[7] Bid shopping also introduces the possibility of ex post regret by the winning prime contractor – if she bids based on expected cost, she might win, then fail to lower her cost, and wind up with negative profit from completing the project ex post. If this might lead to attempts at

[7] In enacting the California Subletting and Subcontracting Fair Practices Act (1941,§4101), the California legislature noted, "The Legislature finds that the practices of bid shopping and bid peddling in connection with the construction, alteration, and repair of public improvements often result in poor quality of material and workmanship to the detriment of the public, deprive the public of the full benefits of fair competition among prime contractors and subcontractors, and lead to insolvencies, loss of wages to employees, and other evils."

renegotiation or breach which were costly to the procurer, this could be a concern; and we show that two obvious candidate policies to prevent it would undo much of the benefit of bid shopping.

The punch line of our paper, therefore, is that if we want to know whether bid shopping is "good or bad," it's complicated, and requires a detailed understanding of the environment. However, it is far from self-evident that the effect is "pernicious." If it involves new subcontractors who would not otherwise have bid, or if it diverts some existing subcontractors in an environment with high bid preparation costs, it likely increases total surplus and leaves fixed, or increases, the likelihood of procurement. Thus, *if* one wants to argue for a ban on bid shopping (on economic rather than ethical grounds), one should argue out of a concern for its distributional effects (or for the solvency of subcontractors or their willingness to participate), its direct costs, its effect on quality and reliability, or the moral hazard it might introduce for winning prime contractors who fail to lower their costs ex post – not out of a mistaken belief that it will make procurement less likely or more costly for procurers.

The rest of our paper proceeds as follows. Section 2 gives a very brief overview of the economic literature on procurement auctions and subcontracting. Section 3 lays out the legal and historical background on bid shopping. Section 4 introduces our model, and characterizes equilibrium play and some comparative statics and preliminary results. Section 5 compares welfare and equilibrium payoffs with bid shopping to those without. Section 6 considers several extensions to our baseline model. Section 7 concludes. Proofs omitted from the text are in the Appendix.

2 Literature on Procurement Auctions

There is a substantial economic literature on procurement auctions. Since none of it deals directly with bid shopping, we give only a brief overview, focusing on issues related to subcontracting.

One branch of literature deals with how prime contractors choose a subcontractor. Wambach (2009) and Nakabayashi (2011) both consider a theoretical model where each prime contractor holds either a first-price or second-price "upstream auction" among a group of potential subcontractors, prior to the "downstream" procurement auction. In this setting, revenue equivalence breaks down: when prime contractors hold first-price auctions, subcontractors have an added incentive to bid more aggressively to help the prime contractor win the prime auction; this incentive vanishes with second-price auctions, since the winning subcontractor's bid does not set the price. Both papers show that first-price upstream auctions therefore lead to lower prices. They also lead to greater efficiency – with second-price upstream auctions, the prime contractor with the lowest-cost subcontractor won't always win the prime auction. Watanabe and Nakabayashi (2011) find that these theoretical predictions hold in a lab experiment: first-price upstream auctions lead to more aggressive bidding than if there was no downstream auction to follow, and lead to greater efficiency than second-price upstream auctions; second-price auctions, however, lead to higher prime contractor profits.

A few papers, mostly empirical, consider the effect of auction design choices in procurement. Branzoli and Decarolis (2015) compare two auction formats used in Italian procurement, "average bid auctions" – where the bidder closest to the average bid wins – and first-price auctions. They find that the switch from average bid to first-price auctions reduced both entry and subcontracting.

(Average bid auctions function like lotteries for a windfall profit, inducing entry even by firms that are not particularly efficient, who then have a stronger incentive to subcontract should they win; first-price auctions limit entry to the more competitive firms, who have less need to subcontract.) Moretti and Valbonesi (2015) compare Italian firms who are fully qualified to complete a project themselves (and can therefore choose whether to subcontract) with firms who are not fully qualified (and are therefore required to subcontract); they find that after controlling for observables, the firms who must subcontract bid higher prices. Marechal and Morand (2003) offer some theoretical analysis on the trade-offs between ex ante and ex post subcontracting (before and after the prime auction), in a setting where there is post-award renegotiation via change orders. Balat, Komarova and Krasnokutskaya (2017) offers insight on the California highway procurement market, and propose a framework for understanding the empirical impact of a choice of ex ante or ex post subcontracting.

A few papers consider the effect of allowing or disallowing subcontracting. Gale, Hausch and Stegeman (2000) consider two firms with non-constant marginal costs, who compete in a sequence of procurement auctions, and can subcontract to each other. With increasing marginal costs, the ability to subcontract increases the firms' profits ex post; however, it also leads to more aggressive bidding, and may hurt the firms ex ante. Marion (2015) similarly considers horizontal subcontracting – firms that compete in the prime auction but also subcontract to other bidders. Theoretically, this can soften competition but also increase efficiency. Analyzing data from California highway construction auctions, he finds evidence that horizontal subcontracting increases efficiency but has a negligible effect on procurement costs. Jeziorski and Krasnokutskaya (2016) model sequential auctions, where the ability to subcontract helps firms manage their capacity constraints. Calibrating a model to the California highway procurement market, they estimate that subcontracting allows a 12% reduction in procurement costs and a 20% increase in the number of projects procured.

A significant strand of literature, starting with Bajari and Tadelis (2001), considers the fact that procurement contracts are incomplete, and subject to post-award revision and renegotiation. Bajari and Tadelis (2001) show that "cost-plus" contracts are preferable to fixed-price contracts when the project is more complex. Bajari, Houghton and Tadelis (2014) use highway paving data to estimate the magnitude of "adaptation costs," and find they account for 8-14% of the winning bid. Bajari, McMillan and Tadelis (2008) examine private sector data to understand the trade-offs between procurement auctions and direct negotiations. Tadelis (2012) finds that the added flexibility in private sector procurement, relative to the restrictions put on public agencies, offers efficiency advantages. De Silva et al. (2017) examine highway procurement data from before and after a policy shift limiting contract revisions, and find that the policy change reduced project costs and improved on-time and on-budget performance. Miller (2014) looks at the impact of contract incompleteness and ex post revisions on both subcontracting and bidding behavior. Gil and Marion (2013) focus on the importance of ongoing relationships between contractors and subcontractors.

A number of empirical papers examine other issues related to procurement auctions. Lewis and Bajari (2011) consider procurement auctions that use scoring rules to favor faster completion of projects, and find this can increase welfare by 22% of the contract's value. Porter and Zona (1993) study bidder collusion in procurement auctions. Krasnokutskaya and Seim (2011) study the impact of “bid preference programs” designed to favor small businesses, domestic firms, or other targeted bidders. Bolotnyy and Vasserman (2019) study scaling auctions, where suppliers submit unit price bids for components of a project, and find this format offers substantial savings. Finally, Li and Zheng (2009), Krasnokutskaya (2011), Balat (2017), and Somaini (2020) use data from procurement auctions to develop or showcase general methodological advances for empirical analysis of auctions, developing tools to account for endogenous entry, unobserved heterogeneity, interdependent costs, or intertemporally linked costs in a dynamic setting.

3 Background on Bid Shopping

3.1 Background – The Subcontracting Process

As noted above, many large construction projects are procured through auctions, and large projects often involve subcontractors doing a substantial part of the work. Prime contractors want to know their costs before bidding in the prime auction, so they seek sub-bids from subcontractors before the auction.

The bid procedure for subcontracts involves two phases. In the first phase, the prime contractor invites subcontract bids, makes available the specifications for the subcontracts, and selects a deadline for the submission of subcontract bids. This deadline is often set just hours before the general contractor’s bid is due (Closen and Weiland, 1980, p. 274). The second or “project buyout” phase occurs only if the prime contractor wins the procurement auction, and consists of the time between the contract award to the prime contractor and the awarding of the subcontracts (Zwick and Miller, 2004, p. 245). During the buyout phase, the prime contractor meticulously reviews the subcontractor bids for any holes or double coverage, and reanalyzes them in relation to the entire project. Indeed, time constraints during the bidding phase leave room for errors, which need to be corrected to ensure a smooth operation of the entire project. Buyout is also the time when purchase orders for materials are issued.

Both the subcontractor bidding stage and the buyout stage present the prime contractor with an opportunity to engage in “bid shopping,” the disclosure of one subcontractor’s bid to another subcontractor in an effort to secure a lower bid. The following passage vividly describes this process during the subcontractor bidding stage:

> Normally, the twenty-four hour period preceding the prime bid deadline is one of great activity, with the subcontractors and suppliers making the rounds of the contractors who are still preparing their bids. By this time the first quotations have generally become known and the quoted prices are often revised downward at the last minute, with the prime contractors revising their bids accordingly.[8]

Indeed, the fear of being bid shopped during the pre award stage has provoked a strategic reaction by subcontractors, who often hold their bids until the last few minutes before the end of the bid

[8] Janke Constr. Co. v. Vulcan Materials Co., 527 F.2d 772, 775-76 (7th Cir. 1976), cited in Closen and Weiland (1980, p. 576).

period. By doing so, subcontractors keep their bid confidential, and limit the opportunity for the prime contractor to communicate with a competing subcontractor and extract a lower price. This phenomenon is reminiscent of “sniping,” the practice of last minute bidding on Ebay that stems from bidders’ fear of being overbid (Roth and Ockenfels, 2002). As a consequence, bid day can be nerve wracking and chaotic, as witnessed in the following description (Korman et al., 1992, p. 26):

> It’s 1:51 p.m. at Swinerton & Walberg Co.’s San Francisco office, and the first telephone price quote from an electrical subcontractor arrives. Then another eight electrical subs call in rapid succession. Nothing unusual here – the general contractor’s bid is due at 2 p.m. In the two hours before the deadline, seven Swinerton & Walberg staff members have fielded about 500 phone calls related to the bid to build a new high school near Fresno. Another 13 compare quotes and scopes until there are only two minutes left. Then numbers are relayed by phone to an employee near the owner’s office. He scribbles them down and runs inside to have the bid stamped – one minute before it’s due.

The hectic pace of bid day inevitably leads to mistakes and oversights that have to be corrected during the buyout stage. The very nature of the buyout process thus requires communication between the prime contractor and his subcontractors, setting the stage for post award bid shopping. Once the general contractor is awarded the prime contract, he is in a strong bargaining position, giving him an opportunity to pressure competing subcontractors to offer a price below the bid of the winning subcontractor.[9] One example of bid shopping is documented in McCandlish Electric, Inc. v. Will Construction Co. (2001). Will Construction was awarded a public contract for renovation on the wastewater treatment plant of the city of Leavenworth, Washington. In the submission of its winning bid, Will relied on McCandlish, who had been the lowest bidder for an electrical subcontract, and in accordance with State law, listed McCandlish as the electrical subcontractor in its bid to the City. Will accepted the City contract, but subsequently asked the City to allow it to substitute Calvert Technologies, since Calvert could perform the electrical work at a lower price. In its finding of fact, the Court determined that Will had engaged in bid shopping.[10]

Alternatively, upon being awarded the prime contract, the general contractor may pressure the winning subcontractor into lowering its original bid by threatening to subcontract the work to a third party (Oertly, 1975, p. 564). This practice is variously known as “bid chopping” or “bid chiseling.” Losing subcontractors who learn the bid price of the winning subcontractor may undercut it in an attempt to secure the job from a prime contractor who is awarded the main contract. This process is known as “bid peddling.” For most purposes, bid peddling is simply a response of competing subcontractors to the bid shopping activity of a general, and bid shopping and peddling may be treated as one (Lambert, 1970, p. 395).

[9]Opportunistic behavior by the main contractor can happen even if there is no bidding for the subcontract. When the Air Force undertook a project to establish installations for detecting and determining the direction and yield of nuclear detonations in North America, General Electic partnered with Air Technology as a subcontractor in submitting a proposal to the Air Force. Air Technologies had considerable experience in the detection of EM radiation, and developped an electromagnetic sensory subsystem for the proposal. The Air Force then sent out a request for proposal to thirty-six companies, including GE. When GE was awarded the contract, and manned with the knowledge it obtained from Air Technologies, it invited competition on the subcontract for the sensor (Air Technology Corp. v. General Electric Co., 199 N.E.2d 538 (Mass. 1964)).

[10]McCandlish Elec. v. Will Constr. Co., 107 Wn. App. 85, 94 (2001).

3.2 Legal Treatment in the United States

What makes bid shopping feasible is that U.S. courts have always held that a subcontractor's bid is only an offer to provide the stated work at this price, and that no actual contract between the two parties is formed until formal acceptance of the subcontractor's bid, which can only happen after the prime contractor becomes the winning bidder on the project. Thus, starting when a subcontractor submits a bid, continuing when they "win" the pre-auction competition for the subcontract and the prime contractor uses that bid to compute and submit his primary bid, and through the time when he wins the primary auction, there is no contractual relation between the subcontractor and the prime contractor. Such a relation exists only after the latter finally accepts the subcontractor's bid (Closen and Weiland, 1980, pp. 565-566; Loulakis and Santiago, 1997).

This absence of contractual liability would seem to put prime contractors at risk, since subcontractors would appear able to withdraw their bid without penalty at any time prior to formal acceptance of their bid. Thus, upon being awarded the project, the prime contractor would find himself unable to complete the project at the anticipated price. Recognizing the possibility of these adverse consequences, starting with Drennan v. Star Paving Co. (1958), courts have relied upon a legal doctrine of "promissory estoppel," viewing the general contractor's use of the winning subcontractor's bid in formulating his bid on the prime as reasonable detrimental reliance. As a consequence, such a subcontractor can be held to his bid, effectively making bid withdrawal impossible (Lambert, 1970, p. 392). This is true even if the subcontractor made a mistake in his sub-bid.[11,12]

Subcontractors often invest significant time in preparing their bids, particularly in mechanical specialties like plumbing, heating, air conditioning and electrical (Schueller, 1960, p. 501). Also, since winning subcontractors can be held to perform on their bid, capacity limitations may prevent them from bidding on other projects, foregoing potential profits elsewhere (Closen and Weiland, 1980, p. 580). However, these costs are not recognized as detrimental reliance, as U.S. courts have universally ruled that upon award of the project the prime contractor is not bound to use the lowest bid subcontractor, even if he used the sub-bid in the calculation of his prime bid, and even if he listed this subcontractor in his proposal (Gregory and Travers, 2010, pp. 30-31).[13] The reason for this disparate treatment stems from contract law: the general contractor cannot accept the subcontractor's offer (bid) "unless and until it knows that its offer has been accepted by the owner" (Gregory and Travers, pp. 30-31).

3.3 Prevalence in the United States

While there is some controversy surrounding the prevalence of the practice,[14] there is clear evidence that bid shopping, and the associated phenomena of bid chopping and bid peddling, are widespread,

[11] For example, in Weitz Company v. Hands, Inc (2016, 294 Neb. 215.), a plumbing subcontractor who refused to perform on his winning subbid of $2,430,000 was found liable, and made to pay $242,000 in damages.

[12] For more details on promissary estoppel and the exact conditions for it to apply see Goldberg (2014, p.1073), Kovars and Schollaert (2006) and Marston et al (2013).

[13] For a review of some of these cases, see Oertly (1975, pp. 557-558).

[14] For example, in a 2015 report to the US Senate, the Government Account Office states that is was "unable to determine if bid shopping occurs or does not occur on federal construction projects" and that it "could not find evidence of bid shopping in the contract files we reviewed."

and that concerns about these practices are first-order. Bid shopping has been, and continues to be, the subject of numerous legal cases brought by both prime contractors and subcontractors – Goldberg (2011, p. 552) makes reference to "more that 60 Drennan-style cases." Attempts to introduce anti-bid-shopping legislation at the federal level date back to 1932 (Schueller, 1960, p. 504). As we'll discuss below, concerns about the effects of bid shopping have led to industry and legislative attempts to curb the practice in nearly every U.S. state, and the passage of state legislation in more than half of them.

Survey research indicates that bid shopping takes place regularly (Arditi and Chotibhongs (2005), Thurnell and Lee (2009)). In a 2008 survey of U.S. and Canadian construction contractors, 80% of respondents knew of others who had engaged in bid shopping or bid peddling, and "an amazing 32% admit(ted) that they has bid shopped or peddled themselves" (Gregory and Travers, 2010, p.29). A survey conducted by the Construction Management Association of America (FMI/CMAA, 2004) identifies bid shopping as one of the most critical issues perceived by owners, architects, engineers, construction managers, general contractors and subcontractors.

Finally, all the professional organizations involved in construction in the US strongly condemn bid shopping, going so far as to call the practice unethical. The Association of General Contractors of America (AGC) is "resolutely opposed" to bid shopping and bid peddling, and in its 1995 guidelines, issued jointly with the American Subcontractors Association (ASA) and the Associated Specialty Contractors, called their use "abhorrent business practices that threaten the integrity of the competitive bidding system." More specifically, the guidelines state:

> The bid amount of one competitor should not be divulged to another before the award of the subcontract or order, nor should it be used by the contractor to secure a lower proposal from another bidder on that project (bid shopping). Neither should the subcontractor or supplier request information from the contractor regarding any sub-bid in order to submit a lower proposal on that project (bid peddling).
>
> The preparation of bids, proposals, submissions or design-build documents is the result of professional consideration which is the intellectual property of the preparer, and so any such information should be considered proprietary. It is unethical to disclose to others, any information that is provided with an expectation that such information will be kept confidential.

The Code of Ethics of the American Society of Professional Estimators (ASPE) prohibits professional estimators from participating in bid shopping, stating that "this practice is unethical, unfair and is in direct violation of this Code of Ethics." The American Institute of Constructors (AIC) credentialing process includes a means of disciplining construction professionals who engage in unethical conduct. Its code of ethics specifically addresses unethical and deceptive practices, including bid peddling and bid shopping. Even some contractors have explicitly banned the practice: "many large general and specialty contractors have developed their own codes of ethics, which their employees are expected to sign" (Mojica and Clarke, 2007, p. 2).

3.4 Legal Treatment and Prevalence in Other Countries

Bid shopping is not limited to procurement in the United States. Court cases reveal that in Canada, *pre*-award bid shopping, and the associated phenomenon of last minute bidding, are prevalent.[15] Post-award bid shopping, however, has effectively been ruled out by a pair of influential court cases. Since R. v. Ron Engineering (1981), Canadian courts have interpreted a general contractor's request for bids to be a contract offer, and the submission of bids by contractors to be an acceptance of that offer, making subcontractors' bids irrevocable, even if they contain an error (Henley, 1991, p. 390). And Ron Engineering also established that when a general contractor uses a subcontractor's bid in his prime bid and names the subcontractor in his bid, a second contract is formed, binding the prime contractor to use this subcontractor if he is awarded the project, so either party is entitled to damages if the other party reneges (Henley, 1991, p. 385).[16] However, the existence of a contract, and hence its breach, is hard to prove if the main contractor does not name the winning subcontractor in his bid, and courts were reluctant to find so,[17] leaving the door ajar for post-award bid shopping. Interestingly, in its best practices guidelines the Canadian Construction Association urges main contractors to name subcontractors in their prime bids (CCA, 2015). Like its US counterpart, this professional organization also condemns bid shopping and bid peddling (ACA, 2008, p. 3):

> All contractors (Prime Contractors and Subcontractors) should not seek nor accept information concerning a competitor's bid prior to bid closing, should they attempt to modify prices after the bid closing. They should avoid any activity that could be construed as bid shopping or bid peddling.

Further confusing things, even after Ron Engineering, courts in some provinces, primarily Alberta and Ontario, ruled that neither the naming of a subcontractor nor carrying its bid requires the winning general contractor to use that subcontractor (Revay, 1997, p. 3), allowing bid shopping to continue. It was not until a second Supreme Court of Canada ruling in Naylor Group v. Ellis Don (2002) that both the prime contractors who seek bids and the subcontractors who bid in the prime auction became in a legally binding relationship. From then on, unless the owner (the procurer) objects, the winning general contractor is contractually bound to use their named subcontractor, and that subcontractor is legally bound to perform the work at their bid price. Post-auction bid shopping in Canada therefore became illegal.

Other Commonwealth countries, including England, Australia and New Zealand, follow the Canadian legal model in which two contracts are formed.[18] The first contract forms when bidders submit

[15] In Gloge v. Northern (1986), Gloge telephoned its bid to Northern only minutes before bid closing, in order to prevent the general contractor from bid shopping among competing subcontractors (Blom, 1987, p. 213) . In Fred Welsh v. B.G.M.(1996) a critical element of Welsh's bid was withheld from BGM until ten minutes before the bid close, in an effort to prevent BGM from bid shopping (Pattison, 2004, p. 738) .

[16] Thus in Peddlesden v. Lidell (1981), Peddlesden submitted a bid which was used by Lidell in his subsequent bid to the owner. When the owner accepted thisb id, Lidell did not wish to use Peddlesden, and substituted another subcontractor. Peddlesden sued and was awarded damages (Henley, 1991, p. 389).

[17] In Ron Brown v Johanson (1990), Brown was the lowest bidder on a mechanical subcontract. When Johanson was awarded the project, he used his own forces to do the mechanical work. Brown sued, arguing that Johanson had used his bid. However, because Brown was not named in Johanson's prime bid, the court refused to find for Brown (Henley, 1991, p. 393).

[18] The defining case for England and Wales is Blackpool & Fylde Aero Club Ltd v Blackpool Borough Council

their offers to the main contractor, and is termed the Process Contract (Mullan, 2012, p. 177). The second contract forms when the main contractor is awarded the project, and is termed the Substantive Contract (Carnie, 2016). Yet case law in these countries is far less developed than in the US or Canada, and it is unclear how courts would handle bid shopping cases. Some commentators have speculated that New Zealand courts would take a dim view on bid shopping,[19] but there is currently no case law to support this stand. Indeed, survey evidence indicates that bid shopping is rampant in New Zealand.[20] In Australia, there is no legislation to prevent bid shopping, and codes of ethics are designed to curb its practice (May et al., 2001, p. 252). All jurisdictions have their own codes, which generally condemn bid shopping. For example, the New South Wales Government Code of Practice for Procurement contains the following passage:[21]

> Principals should not use tender negotiations as an opportunity to trade-off one tenderer's prices against other tenderers' prices in order to obtain lower prices. This practice, known as "bid shopping," is prohibited.

Like their U.S. counterparts, professional organizations take a dim view on bid shopping. For example, in 2009, the Master Building Association of Australia's Code of Ethics contained the following provisions (Uher and Davenport, 2009, pp. 210-211):

> No members shall make known the tender of any subcontractor or supplier before the closing of tenders. It is equally improper for a subcontractor or supplier to disclose his tender to another subcontractor or supplier prior to the closing of tenders ... No member shall attempt to solicit a contract after the closing date of tenders to the disadvantage of a lower tenderer ... Members shall treat all quotations from suppliers of materials and subcontractors as strictly confidential and shall not reveal competitors' prices to each other or to any architect or engineer ... A member who invites a quotation from a supplier or subcontractor and incorporates such price in his tender shall, if the tender is accepted, award the work to the firm who supplied the quotation.

However, despite the presence of codes of ethics and tendering, bid shopping is actively employed by general contractors (Uher and Davenport, 2009, p.211). Indeed, extensive survey evidence by Uher and Runeson (1985), May et al (2001) and London (2005) indicates that bid shopping is endemic to the Australian construction industry, and bid shopping has been implicated in causing high degrees of insolvency among small Australian subcontractors (Coggins et al, 2016, p. 44).

(1990), for Australia it is Hughes Aircraft Systems International v Airservices Australia (1997), and for New Zealand it is Pratt Contractors Ltd v Palmerston North City Council (1995).

[19] Carnie (2016) states that "where an owner ... participates in bid shopping, it might well be found to have breached the terms of the process contract, notwithstanding the existence of a privilege clause."

[20] Thurnell and Lee (2009, p. 1163) found that "bid shopping takes place regularly and is a matter of much concern to subcontractors."

[21] An identical passage is contained in the Victoria's Best Practice Guide for Tendering and Contract Management (2008, p. 26). The Guide also states that it "equally applies to the relationship that principal contractors have with their sub-contractors" (p.8).

3.5 Specific Concerns about Bid Shopping

In condemning bid shopping, contract awarding agencies, associations of general contractors, federations of subcontractors, other professional organizations, and legislatures note a variety of concerns about the practice. One often cited is that it is an attempt by general contractors to enrich themselves at the expense of the other stakeholders. For example, Lambert (1970, p. 395) states that "Any price reductions gained through the use of post-award bid shopping by the general will be of no benefit to the awarding authority." J.C.C., Jr. (1967) concurs, arguing that "the awarding authority receives no benefit since it has already agreed to pay the general a fixed sum."[22]

While correct ex-post, such a view is incomplete, because it ignores any ex-ante endogenous responses of the contracting parties to the presence of bid shopping. Indeed, anticipating a lower post-award cost, the general contractor may submit a lower bid for the prime contract, thereby benefiting the awarding authority. Likewise, in anticipation of being bid-shopped, subcontractors may inflate their pre-auction bids (Gregory and Travers, 2010, p. 32). All else equal, the latter would negatively affect the surplus of the general contractor and the awarding authority alike. In addition, concerns about being bid shopped, and thereby losing bid preparation costs and foregoing profits on other contracts they could have competed for instead, may reduce the number of subcontractors willing to bid on a job. Such a reduced participation rate would raise the general contractors' costs, raising the expected cost to the procuring agency. The welfare effects of bid shopping are therefore complex, and the goal of our paper is to shed some light on this issue.

Another concern often raised relates to quality of the subcontracted work. Armed with the leverage of being awarded the contract, and in an effort to increase profits, a general contractor will approach competing subcontractors during the buyout stage to obtain price concessions; to keep (or win) the job, subcontractors will be tempted to offer or accept substitution of materials or find other ways to cut corners. This is often done under the guise of 'value engineering,' and results in a specification and workmanship slide that ultimately harms the owner, who gets less value for his money and suffers the consequences of eventual failures. In effect, the general contractor "places a profit squeeze on the subcontractors, impairing their incentive and ability to perform to their best, and possibly precipitating bankruptcy."[23] In enacting the California Subletting and Subcontracting Fair Practices Act (1941, §4101), the California legislature explicitly recognized these dangers:

> The Legislature finds that the practices of bid shopping and bid peddling in connection with the construction, alteration, and repair of public improvements often result in poor quality of material and workmanship to the detriment of the public, deprive the public of the full benefits of fair competition among prime contractors and subcontractors, and lead to insolvencies, loss of wages to employees, and other evils.

It should be noted that the concern with subcontractor insolvency is distinct from the problem of low subcontractor bid participation rates. Low participation rates stem from the subcontractor's

[22] The perspective that the only party gaining from bid shopping is the prime contractor engaging in the practice is still prevalent today. For example, the article "Why Bid Shopping Hurts The Masonry Industry," (Masonry Magazine, 2017) states that "the only one who benefits from bid shopping is the (general) contractor" and "There is no benefit seen by the public or the client."

[23] Southern Cal. Acoustics Co. v. C.V. Holder, 71 Cal.2d 719, fn. 7.

desire to cover the fixed cost of bid preparation, whereas bankruptcy occurs because of an inability to cover the fixed overhead of machinery and the central office. The concern over bankruptcy is echoed by Baltz (1997, p. 24), who states that bid shopping "creates lower profit margins for subcontractors," the long-term effect of which may be "a reduction in the number of subcontractors... with a resultant decrease in competition for subcontract work and higher costs."

A final misgiving with bid shopping or bid peddling is that bid-shopped subcontractors must incur the cost of bid preparation, which others can free ride upon. Thus opportunistic subcontractors may refrain from preparing their own bids, essentially waiting until the buyout stage to undercut the lowest bidding subcontractor, and avoid bid preparation costs by relying on the estimating prowess of others (Baltz, 1997, p. 22).

3.6 Attempts to Curb Bid Shopping in the U.S.

Worries about the effects of bid shopping have led to various attempts to curb the practice by industry participants as well as state and federal legislatures.

Contractual Terms

One method that has consistently run into obstacles is contractual provisions. For example, in 2013 the ASA released an updated edition of its popular ASA Subcontractor Bid Proposal, a standard form and accompanying instructions that subcontractors can download from the ASA website and use in their bid proposals (Construction Business Owner, 2013). The revision included a new clause designed to deter general contractors from bid shopping:

> Subcontractor has devoted time, money, and resources toward preparing this bid in exchange for Customer's express agreement that the parties shall have a binding contract consistent with the terms of this bid proposal and Customer unconditionally and irrevocably accepts this bid proposal if it (A) in any way uses or relies on the bid proposal or information therein to prepare "Customer's bid" for the project at issue and Customer is awarded a contract for the work; or (B) divulges the bid or any information therein to others competing with Subcontractor for the work.

However, such unilateral attempts to create a binding contract have not held up in court. For example, in its bid proposal to general contractor West Construction for a public construction project, a Florida asphalt paving subcontractor, Florida Blacktop, included the following provision:[24]

> In the event the "buyer" in any way uses Florida Blacktop, Inc.'s bid . . . such action(s) shall in all instances constitute acceptance of Florida Blacktop, Inc.'s bid and shall create a binding contract between the parties consistent with the bid documents.

West integrated Florida's bid into its own proposal, was awarded the prime contract, and furnished the owner with a list of proposed subcontractors that identified Florida as the asphalt paving

[24] West Construction, Inc. v. Florida Blacktop, Fla. 4th Dist. Ct. App., 2012

subcontractor. Subsequently, West divulged Florida's bid to a third party paving subcontractor and awarded it the work. Florida had no recourse, because West never signed a formal contract with Florida; the court ruled that unilateral attempts to bind the general contractor are invalid, unless the general agrees in advance (Smith, Currie and Hancock LLP, 2013).

On the other hand, if prior to contract award the general contractor were to guarantee the subcontract would go to the lowest bidder, e.g. by signing a letter of intent conditioned only on receipt of the award, then a conditional bilateral contract would be formed and be enforceable (Closen and Weiland, 1980, p. 600).[25] However, a general contractor may be reluctant to commit to rules or otherwise engage in actions that create a conditional bilateral contract, as there are legitimate business reason for not working with the lowest bidder. Because of the hectic pace at which the sub-bids come in right before the general contractor must submit his bid, there are often gaps in the contract with various subcontractors, and the low bidder may turn out to be insufficiently qualified, or not to have the required office personnel or labor force to do the job adequately and in a timely fashion. Making such a deal with an ex-ante trustworthy subcontractor avoids this peril, but deprives the general of legitimate competition that may be needed to secure the prime contract (J.C.C., 1967, p. 1746). For this reason, general contractors will likely be advised by their construction attorneys not to make such promises before or at the time subcontractors submit their bids (Closen and Weiland, 1980, p. 600).

Bid Depositories

With contractual clauses proving ineffective in stopping bid shopping, one might expect industry efforts at self regulation to emerge. One form this has taken is the creation of "bid depositories." A bid depository is a facility created and run by a trade association of construction subcontractors, a bank, or an independent agency, which "receives bids from the subcontractors for the supplying of construction services or supplies and presents those bids *en masse* to the general contractors who intend to bid for the prime contract on a public or private construction job" (Orrick, 1967, p. 520). Use of the registry's services is typically open to all subcontractors and all prime contractors, regardless of their membership status in the registry, provided they agree to abide by its rules. Sanctions for violating those rules include suspension from registry services, expulsion, and circulation of the offender's name (Stewart, 1989, p. 34).

Operation of the registry starts once the awarding authority has announced that it is soliciting bids on a project. The depository then compiles and makes public a list of the prime contractors who indicate a wish to use its services. At their discretion, subcontractors may then deliver sealed bids for each listed prime contractor they wish to engage with, up to a deadline which is usually set three to four hours prior to the prime bid closing time. Once received by the registry, subcontractor

[25] For example, in AROK Construction Co. v. Indian Construction Services (174 Ariz. 291, 1993), prior to submitting its prime bid, Indian Construction successfully convinced AROK to reduce its sub-bid in exchange for a promise to award AROK the subcontract should its prime bid be successful. After winning the prime contract, Indian bid shopped and gave the job to a different subcontractor. AROK sued and the court found in its favor, ruling that Indian's agreement had created a conditional bilateral contract (Kovars, J. and M. Schollaert, 2006, p.10). Similarly, an electrical subcontractor was able to recover lost profits on two subcontracts where it was the lowest bidder, and had been made oral promises by the general contractor that "if we get the job, so do you" (Guarantee Electrical Construction Co. of St. Louis v. HS Construction Company, Mo. App. 2002).

bids may not be amended or withdrawn, or may be withdrawn only under penalty of a fine.

Depository rules typically require that subcontractors using the registry not submit any non-registered bids. Similarly, any prime contractor using the registry may not accept any bids from any subcontractor who did not submit a timely registered bid (Orrick, 1967, pp. 521-522).[26] In effect, this segments the market into two separate competing pools of prime contractors and subcontractors: those who use the registry and those who do not (Stewart, 1989, p. 35). Within the depository pool, bid shopping is thereby effectively ruled out. Indeed, pre-award bid shopping is precluded because the depository rules require subcontractors to submit definitive bids; post-award bid shopping is ruled out because the prime contractors participating in the depository must use depository sub-bids in their prime bids.

Bid depositories have operated in the U.S. since at least the 1930's (Schueller, 1960, p. 528), and at one time were very successful, with more than 100 depositories in operation in the early 1980's (Kaskell, 1981, p. 1). In some sense, they worked too well, and therefore came under antitrust scrutiny. Early cases focused on blatantly anticompetitive practices such as bid rigging, allocation of markets, boycotts and coercion.[27,28] Such practices are restraints of trade, and therefore per se violations of the Sherman Act; the depository institution itself was not on trial, as it merely acted as "an ancillary device to the primary conspiracy" (Stewart, 1989, p. 36). Starting in 1958 with the Bakersfield case,[29] however, state and federal litigation started attacking rules central to the operation of bid depositories, declaring illegal requirements that prime contractors using the depository may not accept outside bids or must report such bids to the registry, stipulations that preclude subcontractors using the depository from submitting bids to outside general contractors, or rules that prohibit generals from accepting subcontractor bids after the deadline of the depository. As a consequence, bid depositories became ineffective in combatting bid shopping or bid peddling, and currently there are only two bid depositories left operating within the United States, the Builders Bid Service of Utah and the Maine Construction Bid Depository.[30] These depositories do not prevent bid shopping, but still have some advantage in that they make bid shopping more apparent, as the bids are public (Carr, 2013, p. 2).

In Canada, bid depositories were formed in the 1950's by the Canadian Construction Association and key trade contractor associations, such as mechanical, electrical and masonry (Read, Kelly and Worthington, 2008, p. 27). The process is initiated by owners (procurers), who may elect to conduct the tender process through the bid depository. After owners submit full project plans to the depository, interested subcontractors then identify their work requirements, and submit their bids prior to a deadline set several days before the end of the prime contractor competition. The depository forwards the subcontractor bids to the owner and the general contractors who expressed

[26] Some depositories allow prime contractors to accept bids from outside subcontractors, but only if they register those bids in the depository themselves. One such example is the Electrical Bid Registration Service of Memphis (Stewart 1989, p. 34).

[27] Schueller (1960, pp. 528-530) contains an exhaustive list of 28 civil or criminal bid depository cases brought by the Department of Justice over the twenty year period 1939-1959.

[28] Two examples of such flagrant violations are the refusal of union labor to subcontractors who do not use the depository (Schueller, 1960, p. 508), and subcontractors eliminating competing bids by paying the competitor's fine for withdrawing his bid (Ducker, 1969, p. 759).

[29] United States v. Bakersfield Associated Plumbing Contractors, U.S. District Court, S.D. California, 1958.

[30] See http://www.buildersbidservice.com and https://agcmaine.org/about/maine-construction-bid-depository/.

an interest in bidding for the project (Coggins, Teng and Rameezdeen, 2016, p. 44). After reviewing the subcontractor bids, prime contractors 'nominate' their subtrades, i.e. select the subcontractors they wish to engage in each trade, and identify these parties in their bids to the owner. The winning prime contractor then must employ the named subcontractors in the construction project (Read, Kelly and Worthington, 2008, p. 27). The Canadian bid depository system effectively precludes all bid shopping after the close of the subcontractor auction. Indeed, Canadian courts have found the winning prime contractor liable for implementing the rules of the depository, in particular the employment of the named subcontractors at their deposited bids. Owners, because of their knowledge of this information and their duty of fair treatment and good faith, are also potentially liable, and have been dragged into court by subcontractors that were bid shopped.[31] Ever since the Canadian Supreme Court decision in Naylor Group v. Ellis Don (2002), bid shopping has been illegal, making it questionable whether the owner should use the bid depository system at all (Read, Kelly and Worthington, 2008, p. 146). As a consequence, bid depositories in Alberta, Ontario, Manitoba and Ottawa effectively ceased operation due to lack of use.[32] Today, bid depositories continue to operate in British Columbia, Saskatchewan and Quebec. In the latter province, use of the bid depository is mandatory for any project whose cost exceeds $20,000.[33]

Federal and State Legislation

The failure of contractual and self-regulatory attempts to curb bid shopping in the U.S. has prompted various attempts at legislative intervention. As early as 1932, bills were introduced in Congress mandating that prime contractors name the subcontractors to be used in their bids to the awarding agency (Schueller, 1960, p. 504). Since 2000, every Congress other than the 116th has had legislation before it making such bid listing mandatory, only to see it die in chambers.[34] Thus, there is essentially no federal law protecting subcontractors from bid shopping. At one time, the Department of the Interior and the General Services Administration required subcontractor bid listing, but they stopped the practice in 1975 and 1983, respectively (GAO, 2015, p. 17). The only protection available to subcontractors comes from the revised Small Business Administration rules implementing the Small Business Job Act of 2010. These rules require that for covered contracts (those that require the submission of a subcontracting plan and whose value exceed a minimal threshold), the prime contractor must notify the contracting officer whenever it does not employ the small business subcontractor used in preparing its bid proposal.[35] Under the current Federal Acquisition Rules, a prime contractor who fails to make a "good faith" effort to use said subcontractor can be found

[31] Twin City Mechanical v. Bradsil, Ontario Court of Justice (1996) and Ken Toby v. B.C. Building Corporation, British Columbia Court of Justice (1997).

[32] See Versace (2008) and Read, Kelly and Worthington (2008, p. 28).

[33] Today, more than 5000 contractors are users of the electronic bid transmission system of the Bureau des Soumissions Déposées du Québec (BSDQ), which reaches an annual volume of more than 4500 projects, attracting some 40,000 bids (BSDQ, Rapport Annuel 2019, p. 4).

[34] Representative Paul Kanjorski introduced "Construction Quality Assurance Acts" in each of the 106th to the 111th Congress (HR4012 (2000), HR1859 (2001), HR1348 (2003), HR2834 (2005), HR3854 (2007) and HR3492 (2009)), Rep. Carolyn Maloney intruduced bills under the same name during the 112th and 113th Congress (HR1778 (2011), HR1942 (2013)), and Rep. Scott Peters introduced "Stop Unfair Bid Shopping Acts" in the 115th and 116th Congress (HR7221 (2018), HR5247 (2019)).

[35] Federal Register, Document No. 2013-16967, 78FR 42391-42406.

to have materially breached its contract, and such failure can be used in the future to evaluate the prime's contract performance.[36]

State legislatures, in contrast, have had considerable success enacting statutes that require listing of subcontractors on public projects. The first such statute was introduced by North Carolina in 1925, and there has been a steady growth of states requiring bid listing since. By 1993, eighteen states already had some sort of listing requirement. Our comprehensive search reveals that as of 2020, that list has grown to twenty-two states, shown in Table 1.[37] A feature common to subcontractor listing laws is that the prime contractor must identify the names of the chosen subcontractors, their bids, and the work to be performed. It usually also includes a procedure for substituting a listed subcontractor, with the awarding authority's approval, for specified reasons. Bid listing laws are not foolproof in preventing bid shopping, however. While the majority of courts have upheld the listed subcontractor's right to perform the subcontract unless statutory reasons for substitution exist, a minority have ruled otherwise (Gregory and Travers, 2010, p. 33). For example, the Supreme Court of Kentucky found that the listing of a subcontractor's name and bid in a general's bid is solely for the protection of the awarding authority, and does not establish a contractual relationship between the named subcontractor and the general contractor (Goldberg, 2011, p. 570). State listing statutes may also limit the listing of subcontractors to certain trades only. For example, Kansas requires only the listing of electrical and mechanical subcontractors.[38] Some listing statutes also leave wiggle room for post-award bid shopping, because they do not require the bid list to be submitted until a specified period after the selection of the winning general contractor. For example, Alaska gives the winning prime five days after its selection to submit its list.[39] Finally, bid listing can be undermined by general contractors who auction off a position on the list to "subcontractors who are willing to perform the work for less than the lowest subbid received" (Daus and Ruprecht, 1990, p. 6).

Since bid listing offers limited protection against bid shopping, other states have come up with alternative remedies. One procurement method that has gained some acceptance is that of "separate specifications" or "multiple primes." Under this system, major subcontracts must be bid separately from the general contract. The oldest such law dates back to 1913, when Pennsylvania enacted its Separations Act, which requires separate first price auctions for plumbing, heating, ventilation, and electrical work in the erection, construction, or alteration of public buildings.[40] New York followed in 1921 with its famous Wicks Law, which mandates separate bidding for plumbing, ventilation and electrical in any building project exceeding a threshold.[41] North Dakota is the only other state where multiple primes is currently mandatory, while Ohio and Texas permit the system but do not require it. Separate specifications make bid shopping impossible, but have been criticized for being costly. Indeed, the system puts more oversight onus on the awarding agency, requiring expertise in dealing with each of the separate trades, as the general contractor has no contractual responsibility to

[36] Federal Acquisition Rules, paragraph 19.704, "Subcontracting Plan Requirements," sections a-12 and a-13

[37] In addition, the state of Wisconsin has a listing requirement for public work by municipalities.

[38] Kansas Statutes Annotated § 75-3741(b)(2) (2018).

[39] Alaska Statutes § 36.30.115 (2018).

[40] Act of May 1, 1913, (P.L. 155, No. 104), as amended by 53 P.S.§1003 (Municipal) and 71 P.S. §1618 (State).

[41] Wicks Law is a collective reference to three separate laws, one that applies to the State (N.Y. State Fin. Law § 135), one that applies to public housing (Public Housing Law, Section 151-a), and one that applies to municipalities and other political subdivisions (General Municipal Law, Section 101).

coordinate their work with the specialists. Furthermore, generals now may find themselves working with subcontractors they have no familiarity or working relationship with. Indeed, concerns about the increased costs associated with the multiple prime system led one commentator to exclaim that Wicks Law is "the poster child of inefficiency in public procurement" (New York City Bar, 2008, p. 5). Preoccupation with inefficiency has led to repeated attempts to repeal or weaken Wicks Law,[42] which so far have failed. Empirical studies comparing the single and multiple prime system have focused on direct procurement costs, and thereby partly measure the effect of eliminating bid shopping, and have obtained somewhat mixed findings (Rojas, 2008).

Table 1: Current regulation of bid shopping by U.S. states

No regulation (21 states)	**Bid listing requirement (22 states)**	**Other regulations (7 states)**
Alabama	Alaska**	*Multiple primes (required)*
Arizona	Arkansas	New York
Colorado	California	North Dakota
Georgia	Connecticut	Pennsylvania
Indiana	Delaware	
Kentucky*	Florida	*Multiple primes (permitted)*
Louisiana	Hawaii	Ohio
Maine	Idaho	Texas
Maryland	Illinois	
Michigan	Iowa	*Filed sub-bids*
Minnesota	Kansas	Massachusetts
Mississippi	Nevada	Wisconsin***
Missouri	New Jersey	
Montana	New Mexico	
Nebraska	North Carolina	
New Hampshire	Oregon	
Oklahoma	Rhode Island	
South Dakota	South Carolina	
Vermont	Tennessee	
Virginia	Utah	
Wyoming	Washington	
	West Viginia	

* Kentucky requires bid listing, but court has ruled this does not create a binding contract
** Alaska: winning prime contractor has five days after selection to submit a subcontractor list
*** Wisconsin also has a bid listing requirement for public work by municipalities

The state of Massachusetts has a procurement system that avoids potential inefficiencies associated with the separate prime system, yet still guards against bid shopping, whether pre- or post-award. Under its "filed sub-bid system," bidding occurs in two stages.[43] First, subcontractors in each of eighteen trades must submit their bids to the awarding authority. Subcontractor bids

[42]See e.g. New York Times, "Pro & Con: Wicks Law; Is Inefficiency Built Into Public Construction?" April 3, 1988.
[43]Massachusetts General Law, Part I, Title XXI, Chapter 149, Section 44F.

may be restricted to certain contractors only, or may be unrestricted. At the end of this stage, the awarding authority compiles a list of all sub-bids received, and makes it available to all general contractors who have expressed interest. In a second stage, general contractors then submit their bids, for every trade including the name of the filed sub-bidder to be used in the work. The winning general must select its listed subcontractors to perform the work at the prices included in its bid. Generals are not required to use the lowest filed sub-bid in the preparation of their own bids. Indeed, at least one court case has ruled that being the lowest filed sub-bidder does not entitle one to the job.[44] However, the awarding authority may request that the general substitute the lowest filed sub-bidder. Massachusetts' filed sub-bid statute dates back to 1939, and for a long time was the only such system. In 2013, Wisconsin became the second state with a filed sub-bid system, but in contrast to Massachusetts, its statute requires prime contractors to use the winning subcontractors in each of the auctions for mechanical, electrical and plumbing work.[45]

State statutes only govern public contracts, leaving subcontractors vulnerable in dealing with private owners as well as federal agencies. Furthermore, as Table 1 shows, only 29 U.S. states offer some form of protection from bid shopping on state contracts, meaning bid shopping is legal in auctions for state contracts in the other 21 states.

4 Model and Equilibrium

4.1 Model

With this background in mind, our aim in this paper is to understand the equilibrium effects of bid shopping. If subcontractors and prime contractors are aware that bid shopping may occur, how does this change their behavior? And on balance, how do the outcomes that result compare to outcomes if bid shopping were impossible?

While bid shopping can occur either before or after the prime auction occurs and the project is awarded, we focus on the latter case – post-award bid shopping. This is largely because submitting sub-bids at the last minute is a well-established and fairly effective countermeasure to pre-award bid shopping, but does not prevent post-award bid shopping. Further, "bid listing" legislation targets post-auction bid shopping; as noted above, such legislation is in effect in roughly half of U.S. states, creating appealing variation for future empirical work once we know "what to look for."

A few key features we want in our model are that subcontractors face a significant cost to prepare a bid; subcontractors who are shown another subcontractor's bid may face a smaller bid preparation cost; and prime contractors can solicit multiple sub-bids before the auction, and may be able to solicit additional ones after. As we'll show, this (along with some simplifying assumptions) leads to a rich yet tractable framework that will help us to understand the equilibrium effects of bid shopping.

Here, then, is our model. There is a procurer – say, a government agency – with a single project to be completed. The procurer values completion of the project at v. The procurer will hold a

[44] Interstate Engineering Corp. v. City of Fitchburg, 367 Mass. 751 (1975), 329 N.E.2d 128

[45] The 2013 biennial budget (Wisconsin Act 20) amended Wisconsin Statute Wis. § 16.855 (Construction Project Contracts) from a multiple primes system to a filed sub-bid system.

second-price auction, with reserve price r. We assume throughout that $r \leq v$.[46] There are m prime contractors eligible to bid.

One part of the project cannot be completed by any of the prime contractors, and must therefore be subcontracted. (Think of a plumbing or electrical component of a construction project, which the winning general contractor will outsource to a specialist firm.) Each prime contractor $i \in \{1, 2, \ldots, m\}$ knows n_i subcontractors who could potentially perform the work. These sets of subcontractors do not overlap – each subcontractor is linked to a single prime contractor. For clarity, we will use female pronouns for prime contractors and male pronouns for subcontractors. Let $N = \sum_{i=1}^{m} n_i$ denote the total number of subcontractors.

Prime contractors face identical costs to perform the portion of the project that is not subcontracted; we normalize this cost to 0. Subcontractors have private costs y_j to perform the subcontracted portion; these costs y_j are independent draws from a probability distribution F. We assume F is continuous and strictly increasing on its support $[\underline{y}, \overline{y}] \subset \mathbb{R}_+$, and admits a density function f. Subcontractors also face an additional cost c (the same for all subcontractors) to prepare a sub-bid; a subcontractor is assumed to know his project cost y_j when deciding whether or not to prepare a bid (and incur the cost c). To avoid trivialities, we assume $r - \underline{y} > c$.

We assume either that bid shopping is impossible (due to legal restrictions or effective self-regulation by the industry), or that it is possible for all prime contractors. That is, we assume that prime contractors do not differ in their ability (or willingness) to bid-shop, and focus on comparing a "world with bid shopping" to a "world without bid shopping."

In the absence of bid shopping, the timing of the game is as follows:

1. Prime contractors (PCs) simultaneously solicit sub-bids from their subcontractors
2. Subcontractors simultaneously decide whether and how much to bid
3. PCs receive their sub-bids, then bid in the prime auction
4. The lowest-bidding PC wins, and the procurer pays her the second-lowest PC bid (or the reserve price r if only one PC submitted a bid)
5. The winning PC pays her lowest-bidding subcontractor his sub-bid; the winning PC and subcontractor deliver the project and incur their costs, and the game ends

If bid shopping is allowed, then the timing is modified as follows:

1. PCs simultaneously solicit sub-bids from their pre-auction subcontractors
2. Pre-auction subcontractors (with correct beliefs about what will happen after the auction) simultaneously decide whether and how much to bid
3. PCs receive these sub-bids, then (with correct beliefs about what will happen after the auction) bid in the prime auction

[46] Without bid shopping, as in Samuelson (1985), social surplus is maximized at $r = v$ and procurer surplus is maximized at $r < v$.

4. The lowest-bidding PC wins, and the procurer pays her the second-lowest PC bid (or the reserve price r if only one PC bid)

5. The winning PC approaches one or more new subcontractors (who did not bid pre-auction) in an attempt to lower her costs, and shows them her lowest pre-auction sub-bid. We'll explain the exact details of how we model bid shopping below. Bid shopping results in one of three possible outcomes: the winning prime contractor replaces the subcontractor, sticks with the same subcontractor at a lower price, or sticks with the same subcontractor at the pre-auction bid price.

6. The winning PC pays a subcontractor (either the pre-auction low bidder or the post-auction replacement), the winning PC and this subcontractor deliver the project and incur their costs, and the game ends

Throughout the paper, we'll assume that the number of pre-auction subcontractors n_i each prime contractor has access to is common knowledge. We assume that subcontractors know their costs before deciding whether to incur the entry cost and bid, and that pre-auction subcontractors do not observe each others' entry decisions before bidding. Since the prime auction is a second-price auction with private costs, we assume prime contractors play the dominant-strategy equilibrium of bidding their costs (or their expected costs if there is post-auction uncertainty).

4.2 Benchmark: Equilibrium Play in the Absence of Bid Shopping

In the game without bid shopping, each prime contractor's bid in the prime auction is her cost, which is equal to the lowest sub-bid she received before the auction. Thus, the lowest sub-bid determines the winning prime contractor, and the subcontractor submitting that bid therefore gets paid his bid and delivers the subcontracted portion of the project. From the subcontractors' point of view, then, the game is equivalent to a symmetric, N-bidder first-price auction with an entry cost. Since subcontractors know their costs when they make entry decisions, the game among the subcontractors has a symmetric equilibrium where each subcontractor enters if his cost is below a common threshold level.

Proposition 1 *Define y^* as the unique solution to*

$$(r - y^*)(1 - F(y^*))^{N-1} \quad = \quad c \tag{1}$$

and define a function $\beta : [\underline{y}, y^*] \to [0, r]$ by

$$\beta(y) \quad = \quad y + \frac{1}{(1 - F(y))^{N-1}} \left(c + \int_y^{y^*} (1 - F(s))^{N-1} ds \right) \tag{2}$$

If bid shopping is not allowed, the game among subcontractors has a unique[47] *symmetric equilibrium, in which subcontractors with costs $y > y^*$ do not bid, and subcontractors with costs $y \leq y^*$ bid $\beta(y)$.*

[47]Uniqueness is up to the actions of subcontractors with cost realization exactly y^*, who are indifferent between entering (and bidding $\beta(y^*) = r$) and staying out.

From (1), we can see that the entry threshold y^* lies within the support of F and below $r - c$. Since the left-hand side of (1) is increasing in r and decreasing in y^* and N, y^* is increasing in r and decreasing in N and c.

Proof of Proposition 1. Given our assumption that prime contractors will bid their costs, we verify this is an equilibrium by ruling out deviations by subcontractors. Note that $\beta(y^*) = r$, so any deviation above the range of $\beta(\cdot)$ cannot induce a general contractor to bid; usual arguments rule out deviations to bids below $\beta(\underline{y})$.

For a subcontractor with cost y, a bid of r gives expected surplus

$$(r - y)(1 - F(y^*))^{N-1} - c$$

since it will win if and only if no other subcontractor bids, which occurs in equilibrium with probability $(1 - F(y^*))^{N-1}$. Comparing this to (1), it is positive for $y < y^*$ and negative for $y > y^*$.

Since β is continuous and we've ruled out deviations above and below the range of β, any plausible deviation is to the equilibrium bid of some type of subcontractor y'. A subcontractor with cost y who bids $\beta(y')$ earns expected payoff

$$U(y, \beta(y')) \quad = \quad (1 - F(y'))^{N-1} (\beta(y') - y) - c$$

Plugging in (2) and simplifying,

$$U(y, \beta(y')) \quad = \quad (1 - F(y'))^{N-1}(y' - y) + \int_{y'}^{y^*} (1 - F(s))^{N-1} ds$$

Differentiating and simplifying,

$$\frac{\partial}{\partial y'} U(y, \beta(y')) \quad = \quad (N - 1)(1 - F(y'))^{N-2} f(y')(y - y')$$

so U is increasing in y' when $y' < y$, decreasing when $y' > y$. For $y \leq y^*$, then, $U(y, \beta(y)) \geq U(y, \beta(y'))$ for all $y' \neq y$, including $U(y, \beta(y)) \geq U(y, \beta(y^*)) = U(y, r) \geq 0$, so entering and bidding $\beta(y)$ is a best-response. For $y > y^*$ and any $y' \leq y^*$, $U(y, \beta(y')) \leq U(y, \beta(y^*)) < 0$, so not bidding is a best-response. Thus, the strategies described in Proposition 1 are an equilibrium.

The proof that this is the *only* symmetric equilibrium is in the appendix. □

Throughout the paper, we will focus on, and make reference to, "symmetric equilibrium." We are assuming prime contractors play their weakly dominant strategy, and when bid shopping is allowed, subcontractors bidding after the prime auction similarly have a unique best response. In our context, "symmetric" therefore refers to all subcontractors who might bid *before* the prime auction ("*pre-auction subcontractors*") playing the same strategy. Without bid shopping, the symmetric equilibrium above is the only equilibrium in which all pre-auction subcontractors enter with positive probability.[48]

[48] Asymmetric equilibria involve a subset of subcontractors being "inactive" (not entering regardless of their realized

4.3 Comparative Statics Without Bid Shopping

Next, we briefly explore a few consequences of our model in the benchmark game without bid shopping, which will help to simplify the analysis or prove useful when we compare outcomes to those reached with bid shopping.

Each prime contractor will solicit bids from all her subcontractors

First, we note that prime contractors have incentives to solicit bids from as many subcontractors as possible:

Proposition 2 *A prime contractor's expected surplus is increasing in the number of subcontractors she solicits bids from.*

The proof is in the appendix. Focusing on prime contractor 1 (PC1), we show that an increase in the number of subcontractor bids PC1 solicits before the auction increases the likelihood she receives at least one sub-bid; decreases the likelihood the opposing PCs receive sub-bids; shifts the distribution of PC1's costs toward lower values; and shifts the distribution of the other PCs' costs (and therefore bids) toward higher values. All of these increase PC1's expected payoff.

As a result of Proposition 2, we do not think of the prime contractors as facing a strategic question of how many of their subcontractors to approach for bids; we assume each PC solicits bids from all n_i of her subcontractors. (Outside of the proof of Proposition 2, we therefore do not distinguish between the number of subcontractors a prime contractor has access to and the number of bids she solicits.)

Total surplus is maximized at $r = v$

We can calculate expected total surplus as follows. If no subcontractor enters, the project is not procured and total surplus is 0. If any subcontractor enters, the project is procured, and total surplus is $v - y - \tilde{N}c$, where $\tilde{N}$ is the number of subcontractors who entered and y is the actual cost of the subcontractor who does the work.[49] In equilibrium, this is the subcontractor with the lowest cost, if (and only if) the lowest-cost subcontractor has cost below y^*; otherwise, nobody enters. The CDF of the lowest of N independent draws from F is $1-(1-F(s))^N$ which has density $N(1-F(s))^{N-1}f(s)$; and since each subcontractor enters if his cost is below y^*, the expected value of $\tilde{N}$ is $NF(y^*)$. We can therefore calculate total surplus as

$$W = \int_{\underline{y}}^{y^*} (v-s)N(1-F(s))^{N-1}f(s)ds - NF(y^*)c \tag{3}$$

which leads to:

costs), and such asymmetric equilibria exist in some environments but not others. In the game with bid shopping considered later in this section, the scope for asymmetric equilibria is larger, but we continue to focus on symmetric equilibria.

[49] The procurer's surplus is $v - P$, where P is the price paid to the winning prime contractor; the winning prime contractor's surplus is $P - b$, where b is his payment to the subcontractor performing the job; the winning subcontractor earns $b - y - c$ net of entry costs; and the $\tilde{N} - 1$ subcontractors who bid and lost each earn a surplus of $-c$.

Proposition 3 *In the absence of bid shopping, total surplus is single-peaked in r, and maximized at $r = v$.*

Proof of Proposition 3. The reserve price r does not appear explicitly in the statement for W, but enters through its impact on y^*, which is increasing in r. Thus,

$$\begin{aligned}
\frac{dW}{dr} &= \frac{\partial W}{\partial y^*}\frac{\partial y^*}{\partial r} = \left[(v-y^*)N(1-F(y^*))^{N-1}f(y^*) - Nf(y^*)c\right]\cdot\frac{\partial y^*}{\partial r} \\
&\propto Nf(y^*)\left[(v-y^*)(1-F(y^*))^{N-1} - c\right] \\
&= Nf(y^*)\left[(v-y^*)(1-F(y^*))^{N-1} - (r-y^*)(1-F(y^*))^{N-1}\right] \\
&= Nf(y^*)(1-F(y^*))^{N-1}(v-r)
\end{aligned}$$

Thus, total surplus is increasing in r when $r < v$, and decreasing when $r > v$. □

The effect of N on total surplus is ambiguous

Note from (3) that total surplus depends only on the total number of subcontractors N, not on how they are distributed across prime contractors $(n_1, n_2, \ldots, n_m)$. Perhaps surprisingly, having more subcontractors can either increase or decrease total surplus, depending on the details of the environment.

Proposition 4 *In the absence of bid shopping, the effect of N on total surplus is ambiguous. For example, for $r \leq v$...*

1. *Total surplus is increasing in N if c is sufficiently small*
2. *Total surplus is decreasing in N if $f(\underline{y}) > \frac{1}{r-\underline{y}}$ and c is sufficiently large*

The proof is in the appendix. For intuition, the probability of procurement, $1-(1-F(y^*))^N$, is decreasing in N if and only if $\frac{f(y^*)}{1-F(y^*)} \geq \frac{1}{r-y^*}$. (An increase in N means more chances to get a low realization of y, but also a lower entry threshold y^*; when $\frac{f(y^*)}{1-F(y^*)} \geq \frac{1}{r-y^*}$, the latter effect outweighs the former.) If c is very large, y^* is close to $\underline{y}$, so the probability of procurement is decreasing if $f(\underline{y}) > \frac{1}{r-\underline{y}}$. Further, when y^* is close to $\underline{y}$, any subcontractors who enter will have nearly identical costs, so there's little additional value in getting more shots at an even-lower cost, so total surplus decreases in N. On the other hand, for c very small, the entry threshold y^* is close to the smaller of r and $\overline{y}$, and therefore does not respond much to a change in N; the effect of one more chance at a low cost realization outweighs the marginal reduction in y^*, and an increase in N increases surplus.

For an illustrative example, let F be the uniform distribution on $[0, 1]$, and let the reserve price r match the procurer's value v. Table 2 shows the entry threshold y^* and total surplus W for four combinations of r, c and N, to illustrate that an increase in N can either increase or decrease total surplus. Moving from setting (1) to (2), the increase in N increases total surplus; moving from (3) to (4), the increase in N decreases social surplus.

Table 2: The effect of N on total surplus is ambiguous

setting	(1)	(2)	(3)	(4)
r and v	0.5	0.5	10	10
c	0.1	0.1	0.3	0.3
N	3	4	3	4
y^*	0.2974	0.2566	0.8192	0.6819
surplus	0.1514	0.1684	8.9587	8.8870

For intuition, when $r = v = 0.5$, most of the social surplus is generated when some subcontractor gets an extremely low cost realization, and the increase in N increases the likelihood of at least one such draw. When $r = v = 10$, welfare is large any time there's an entrant, and the increase in N increases expected entry costs paid but also increases the likelihood that nobody enters (by worsening the "coordination problem" among subcontractors), which together dominate the gains from a reduction in the expected cost of the lowest-cost subcontractor.

Thus, depending on the environment, total surplus can either increase or decrease with N, the total number of subcontractors available pre-auction. When we get to our welfare analysis, we will first compare total surplus with and without bid shopping when the number of pre-auction subcontractors is the same across the two cases. Proposition 4 will help to extend the analysis to the (perhaps more likely) event that allowing bid shopping would reduce the number of subcontractors bidding before the auction, by diverting one or more of these subcontractors to post-auction bidding.

4.4 Defining "Simple" Post-Auction Bid Shopping

Next, we consider the game when post-auction bid shopping is possible. We will use "incumbent subcontractor" to refer to the subcontractor who made the lowest pre-auction sub-bid to the winning prime contractor, i.e., the subcontractor who will do the work if the winning prime contractor does not find a new subcontractor to replace him. We suppose that after the auction, the winning prime contractor will approach one or more new subcontractors (*not* from among those who bid before the auction), show them the incumbent's sub-bid, and solicit additional bids. Subcontractors approached after the auction have project costs drawn independently from a continuous distribution F_0, which need not be the same as F. We allow for the possibility that the subcontractors preparing bids after the auction might have lower bid preparation costs, since they are being shown the incumbent's bid and may be able to free ride on some of his bid preparation effort; specifically, we assume the new subcontractors have bid preparation costs αc, with $\alpha \in [0, 1]$.

In this section, we consider a special case where the winning prime contractor approaches exactly one new subcontractor;[50] we will generalize in a later section. When the winning prime contractor shows the incumbent's sub-bid b to a new subcontractor, the latter correctly infers the incumbent's actual cost. (In equilibrium, this is the lowest of the costs of all the pre-auction subcontractors, so we refer to it as y_{min}.) The post-auction subcontractor also knows the realization of his own

[50]Having the winning prime contractor find a new subcontractor to approach with probability less than one is easily incorporated by letting the distribution of post-auction subcontractor costs F_0 have a point mass at $+\infty$.

cost y_0, and faces bid preparation cost αc. Should this subcontractor offer a bid below b, the prime contractor will return to the incumbent to give him a chance to beat the new bid, then return to the new bidder with another chance to undercut the incumbent, and so on, simulating a second-price auction between the incumbent and the new subcontractor. Since this is all anticipated...

- if $y_0 > y_{min} - \alpha c$, the new subcontractor does not expect to be able to outbid the incumbent and still cover his bid preparation costs, so he does not bid; the winning PC therefore honors the incumbent's pre-auction bid
- if $y_0 < y_{min} - \alpha c$, the new subcontractor enters and undercuts b, leading to a bidding war between the new subcontractor and the incumbent, which the new subcontractor "wins" at the incumbent's actual cost level y_{min}

Recall that y^*, defined by (1), is the entry threshold in the absence of bid shopping. We assume that $F_0(y^* - \alpha c) < 1$ – specifically, that the post-auction subcontractor does not have too large a cost advantage over the pre-auction subcontractors.[51] We also assume that $F_0(y^* - \alpha c) > 0$, since otherwise the threat of bid shopping would have no effect. We allow for any $\alpha \in [0, 1]$.[52]

The assumptions that only one new subcontractor is approached, and that the competition between the incumbent and the new bidder approximates a second-price auction, together imply that when the incumbent subcontractor is replaced, the new subcontractor's bid (the price paid by the prime contractor) is exactly the incumbent's actual cost level. This detail will simplify our analysis, giving us benchmark results from which later results will depart; as a result, we refer to this form of bid shopping as "simple bid shopping."

4.5 Equilibrium Play with Simple Bid Shopping

Under simple bid shopping, on the equilibrium path, the prime contractor will infer a subcontractor's cost level y from his bid b, and will correctly calculate her expected cost should she win,

$$C(b, y) \quad = \quad (1 - F_0(y - \alpha c))b + F_0(y - \alpha c)y \tag{4}$$

since she will pay y if the new subcontractor enters and undercuts the incumbent, and b if the new subcontractor has too high a cost to enter. We can verify that given the equilibrium bid function $b(\cdot)$ we find later, $C(b(y), y)$ is strictly increasing in y; since pre-auction subcontractor bids will be monotone in subcontractor costs, the prime contractor will "accept" the lowest sub-bid she receives, and bid her inferred expected cost $C(b, y)$. Should she win, the subcontractor who submitted the

[51] If $F_0(y^* - \alpha c) = 1$, then any pre-auction subcontractor with costs sufficiently close to y^* is sure to be successfully bid-shopped, and will not enter in equilibrium. As long as $F_0(\underline{y} - \alpha c) < 1$, a symmetric equilibrium with entry still exists, but the entry threshold and other implications are different from those considered below. We focus on the case $F_0(y^* - \alpha c) < 1$, since $F(y^* - \alpha c) < F(y^*) < 1$ and we find it implausible that the post-auction subcontractors would have a large cost advantage over the pre-auction subcontractors.

[52] When $\alpha = 0$, we think of it as the limit case $\alpha \downarrow 0$, so that the new subcontractor will still choose not to enter when he believes he is higher-cost than the incumbent and will therefore be outbid if he enters. Thus, on the equilibrium path, the incumbent always receives either his original pre-auction bid b or nothing. If we instead assumed $\alpha = 0$ and the post-auction subcontractor would always enter, this would not change the qualitative conclusions, just some details in the proofs.

low pre-auction bid b will earn $b - y$ if the post-auction subcontractor has cost above $y - \alpha c$ and doesn't enter, and 0 otherwise, for an expected profit (gross of bid preparation costs) of

$$V(b, y) \quad = \quad (1 - F_0(y - \alpha c))(b - y) \tag{5}$$

Together, (4) and (5) give the result that in equilibrium,

$$V(b, y) \quad = \quad C(b, y) - y \tag{6}$$

In words, a subcontractor's expected surplus from "winning" the pre-auction competition matches the prime contractor's expected cost induced by his bid, minus the subcontractor's own cost. This holds trivially in the absence of bid shopping – the subcontractor's surplus from submitting the low sub-bid is $V(b, y) = b - y$, and the prime contractor's cost $C(b, y)$ is simply b. That (6) holds with bid shopping, however, is not obvious: it says that if a subcontractor with cost y submits a bid b which induces the prime contractor to bid B (given correct beliefs about the subcontractor's true cost), the subcontractor's expected payoff from winning the pre-auction competition is $B - y$. In a limited sense, this will imply that the subcontractor's pre-auction "bid inflation" (due to the expectation of being bid-shopped ex post) and the general contractor's "bid deflation" (due to the anticipated cost reduction from bid shopping) will cancel each other out. In particular, it means that in a symmetric, monotone equilibrium, if the threshold type induces a prime contractor bid of r, he receives an expected payoff of $r - y$ in the event his bid is the low pre-auction bid – exactly the same as without bid shopping. For this reason, with "simple" bid shopping, there is a symmetric equilibrium with the same entry threshold y^* as the symmetric equilibrium without.

Proposition 5 *Let y^* continue to refer to the participation threshold without bid shopping, that is, the solution to* (1)*. With simple bid shopping as described above, there is a symmetric equilibrium with the same participation threshold y^*; subcontractors with costs $y \leq y^*$ bid*

$$b(y) \quad = \quad y + \frac{c + \int_y^{y^*} (1 - F(s))^{N-1}(1 - F_0(s - \alpha c))ds}{(1 - F(y))^{N-1}(1 - F_0(y - \alpha c))} \tag{7}$$

and subcontractors with costs $y > y^*$ stay out.

The proof is in the appendix. The key insight is that the participation threshold for pre-auction subcontractors is unchanged; this follows from the fact, noted above, that if a subcontractor with cost y makes an equilibrium bid b which leaves the general contractor with expected costs r, the subcontractor's own expected payoff is $(1-F(y))^{N-1}(1-F_0(y-\alpha c))(b(y)-y) = (1-F(y))^{N-1}(r-y)$, and that this is therefore equal to c at exactly $y = y^*$, the entry threshold without bid shopping. The equilibrium bid function then follows from the envelope theorem, which is again equivalent to incentive compatibility.[53]

[53] There is one additional complication that must be handled in the proof: a subcontractor with cost y who "imitates" a different cost level y' pre-auction can still compete post-auction as his true type, altering the "deviation payoffs" that must be considered. We account for this in the proof in the Appendix.

As we discuss later, there are other symmetric equilibria as well, but (as we'll explain) the other symmetric equilibria all fail a common equilibrium refinement and strike us as less plausible than this one. We view this equilibrium as the focal one, and therefore use it below to compare equilibrium outcomes with bid shopping to equilibrium outcomes without; but we do note that for total surplus, this equilibrium is the "best" equilibrium with bid shopping, so our results below would be less positive with a different (symmetric) equilibrium selected.

4.6 Impact of Bid Shopping on Bidding Behavior

We can compare the equilibrium in Proposition 5 to the equilibrium of the game without bid shopping (Proposition 1), to find the "equilibrium effect of bid shopping." The fact that the participation threshold y^* is unchanged makes the comparison very clean. (This means we are comparing bidding behavior with and without bid shopping while holding fixed the number of subcontractors bidding pre-auction; in the next section, we will consider the effect when bid shopping diverts one or more subcontractors from pre-auction to post-auction bidding.)

We begin by examining the effect bid shopping has on the bidding behavior of subcontractors.

Proposition 6 *As above, let β refer to the subcontractors' equilibrium bid function without bid shopping, and b their equilibrium bid function with bid shopping. Then (i) $b(y^*) > \beta(y^*)$, (ii) $b(y) < \beta(y)$ whenever $F_0(y - \alpha c) = 0$, and (iii) $b(y) - \beta(y)$ is either everywhere positive or crosses zero once from below.*

The presence of bid shopping has two effects on the bid behavior of the subcontractors. On one hand, bidders face increased competition for winning the subcontracted work, which makes them bid more agressively. On the other hand, the higher their cost, the more likely they will be bid shopped and therefore not be able to recoup their bid preparation cost c. To guard against this event, subcontractors wish to inflate their bid. Which force has the upper hand depends upon the subcontractor's cost: if y is sufficiently high then the bid inflation effect dominates, and if y is sufficiently low then the competitive effect dominates. To see why, consider the highest cost subcontractor to participate in the bidding, type y^*. In the absence of bid shopping, this subcontractor bids the reserve price r and receives zero expected surplus. When bid shopping is present, the same subcontractor type faces a positive likelihood of being bid shopped, so to avoid earning negative surplus must inflate his bid above the reserve, so we have $b(y^*) > \beta(y^*)$. Meanwhile, a subcontractor with sufficiently low cost will never be bid shopped ($F_0(y - \alpha c) = 0$), but still faces a competitive effect because subcontractors with costs above his are bidding more aggressively, so $b(y) < \beta(y)$ when y is sufficiently low.

Next, we consider the effect of bid shopping on the general contractor's bidding behavior in the prime auction. Despite the ambiguous impact of bid shopping on the bid behavior of her subcontractors, the general contractor bids more aggressively due to the prospect of successfully reducing her cost after the auction:

Proposition 7 *Let y be the lowest realized cost of a prime contractor's subcontractors. Then in the presence of bid shopping, the prime contractor bids strictly less than $\beta(y)$, for all $y < y^*$.*

Proof. With bid shopping, the prime contractor has a realized cost of y whenever the new subcontractor enters, and a cost of $b(y)$ otherwise. Thus the prime contractor bids her expected cost

$$\begin{aligned}(1-F_0(y-\alpha c))b(y)+F_0(y-\alpha c)y &= y+\frac{c+\int_y^{y^*}(1-F(s))^{N-1}(1-F_0(s-\alpha c))ds}{(1-F(y))^{N-1}} \\ &< y+\frac{c+\int_y^{y^*}(1-F(s))^{N-1}ds}{(1-F(y))^{N-1}} \quad = \quad \beta(y),\end{aligned}$$

where strict inequality follows because by assumption $F_0(s-\alpha c)>0$ for s close to y^*. □

5 Welfare Effects of Bid Shopping

Next, we consider the effect of bid shopping on surplus. A key question in this exercise is whether the possibility of bid shopping diverts some subcontractors from pre-auction bidding to post-auction bidding, or whether the subcontractors approached after the auction would not have bid before the auction anyway. The first possibility – that if bid shopping is allowed, fewer subcontractors will be available before the auction since some may wait to bid after – may be the more natural assumption; but we will consider the latter possibility first, since the analysis is simpler.

5.1 Bid Shopping Without "Diversion"

We first consider the case where the number of subcontractors active before the auction is the same whether or not bid shopping is allowed – that is, we compare total surplus when there are N subcontractors and no bid shopping, to total surplus when simple bid shopping occurs but there are still N subcontractors active before the auction.

Theorem 1 *If bid shopping does not divert subcontractors from the pre-auction competition, then bid shopping increases total surplus.*

Proof. As before, let y_{min} be the lowest of the pre-auction subcontractors' realized costs, and $\tilde{N}$ the number of pre-auction cost realizations below y^*. We noted above that without bid shopping, the total realized surplus is $\mathbf{1}_{y_{min}\leq y^*}(v-y_{min})-\tilde{N}c$. With simple bid shopping, this is also the combined surplus realized by the procurer, the general contractors, and the pre-auction subcontractors, i.e., the surplus realized by everyone but the post-auction subcontractors. This is because with bid shopping, either the lowest-bidding pre-auction subcontractor is not replaced (in which case ex post surplus is the same as without bid shopping), or he is replaced, in which case the procurer earns $v-P$, the winning prime contractor earns $P-y_{min}$, and $\tilde{N}$ pre-auction subcontractors earn $-c$, for combined payoffs $v-y_{min}-\tilde{N}c$. (To put it another way, when the incumbent subcontractor is replaced after the auction, since the new subcontractor gets paid the incumbent's cost y_{min}, the incumbent's loss $b(y_{min})-y_{min}$ is exactly equal to the prime contractor's gain.) Combined expected surplus of everyone but the post-auction subcontractors is therefore the same with and without bid shopping, and total surplus therefore increases by the expected surplus of the post-auction subcontractors. □

Thus, bid shopping increases total surplus by exactly the surplus earned by the post-auction subcontractors. To calculate this surplus, and thus the increase in surplus due to bid shopping, fix y_{min}. It's easiest if we think of all prime contractors using the same post-auction subcontractor; the surplus we calculate will in fact be the combined ex ante surplus of all the post-auction subcontractors.

If $y_{min} > y^*$, the auction fails, and there is no surplus available to the post-auction subcontractor. If $y_{min} \leq y^*$, the auction succeeds, and the winning contractor approaches the post-auction sub, who has cost $y_0 \sim F_0$. If $y_0 > y_{min} - \alpha c$, this subcontractor will choose not to enter (correctly inferring y_{min} from the equilibrium bid and knowing he can't recover his entry cost by bidding against the incumbent). If $y_0 < y_{min} - \alpha c$, on the other hand, the post-auction subcontractor will enter and win, earning $y_{min} - y_0 - \alpha c$. Taking the expectation over y_0, the post-auction subcontractor's expected surplus given y_{min} is

$$\int_0^{y_{min}-\alpha c} (y_{min} - s - \alpha c) dF_0(s) \quad = \quad \int_0^{y_{min}-\alpha c} F_0(s) ds$$

(the latter by integration by parts). Taking the expectation over y_{min}, whose CDF is $1-(1-F(\cdot))^N$, we can write the ex ante surplus of the post-auction subcontractors – and thus the increase in total surplus from bid shopping – as

$$\Delta W \quad = \quad \int_{\underline{y}}^{y^*} \left[\int_0^{y_{min}-\alpha c} F_0(s) ds \right] N(1 - F(y_{min}))^{N-1} f(y_{min}) dy_{min} \tag{8}$$

A few observations follow directly from (8):

Proposition 8 *With simple bid shopping without diversion...*

1. *Total surplus is decreasing in α (the entry costs of the post-auction subcontractor)*

2. *Total surplus increases with a shift in the post-auction subcontractors' cost distribution F_0 toward either lower costs (via first-order stochastic dominance) or more varied costs (via a mean-preserving spread)*

3. *The gain in surplus from bid shopping is decreasing in c*

Proof of Proposition 8. Note that $W_{shop} = W_{noshop} + \Delta W$ (where W_{shop} and W_{noshop} are the total surplus with and without bid shopping, respectively), and W_{noshop} and y^* were determined without reference to bid shopping and therefore do not depend on α or F_0. An increase in α shrinks the region of integration of the inner integral of ΔW, decreasing ΔW and therefore W_{shop}. A first-order shift toward lower costs increases F_0 pointwise, increasing the inner integral; a mean-preserving spread applied to F_0 increases the value of $\int_0^x F_0(s)ds$ for every x, again increasing the inner integral and therefore W_{shop}. Since y^* is determined by $(r - y^*)(1 - F(y^*))^{N-1} = c$, y^* is decreasing in c; ΔW is increasing in y^*, and decreasing directly in c (which appears only in the upper limit of the inner integral) as well, so an increase in c decreases ΔW. $\square$

5.2 Bid Shopping With "Diversion"

Next, we consider the possibility that when bid shopping is allowed, some subcontractors will choose not to bid before the auction and will instead wait to be approached by the winning prime contractor after the auction. We label this "diversion." We continue to assume that the winning prime contractor will approach only a single subcontractor who did not already bid. To make the comparison relevant, we assume $F_0 = F$ – bidding before or bidding after the auction does not affect a subcontractor's cost to complete the project, although it may affect their bid preparation cost (α may be less than 1).

Theorem 2.a *Suppose that $r \leq v$. If $f(\underline{y}) > \frac{1}{r-\underline{y}}$ and c is sufficiently large, then bid shopping that diverts one or more subcontractors from pre-auction bidding increases total surplus.*

This follows directly from Proposition 4 part 2 and Theorem 1. Under the conditions given, reducing the number of pre-auction subcontractors on its own increases surplus; then allowing bid shopping (at the new number of pre-auction subcontractors) increases it further.

Theorem 2.b *Suppose that $r \leq v$. If c is sufficiently small...*

1. *Bid shopping that diverts exactly one subcontractor from pre-auction bidding increases total surplus if $\overline{y} < r$, and decreases total surplus if $\overline{y} > r$.*
2. *Bid shopping that diverts two or more subcontractors from pre-auction bidding reduces total surplus.*

5.3 Who Gains and Who Loses from Bid Shopping

The results above concerned the effect of bid shopping on total surplus; next, we investigate how those gains and losses are distributed.

Theorem 3.a *Simple bid shopping without diversion makes pre-auction subcontractors worse off. Simple bid shopping that diverts one or more subcontractors makes the remaining pre-auction subcontractors better off.*

Proof. Let N be the total number of subcontractors, and M the number diverted. Without bid shopping, a subcontractor with cost y earns expected surplus

$$U_{noshop}(y) \quad = \quad \mathbf{1}_{y \leq y_N^*} \int_y^{y_N^*} (1 - F(s))^{N-1} ds$$

where y_N^* refers to the entry threshold given N subcontractors. With bid shopping and $N - M$ remaining pre-auction subcontractors, the payoff to each one can be calculated via the envelope theorem as

$$U_{shop}(y) \quad = \quad \mathbf{1}_{y \leq y_{N-M}^*} \int_y^{y_{N-M}^*} (1 - F(s))^{N-M-1} (1 - F(s - \alpha c)) ds$$

When $M = 0$ (no subcontractors are diverted), the latter integrand is smaller, and the two integrals are taken over the same range, so $U_{shop}(y) \leq U_{noshop}(y)$, with strict inequality for $y < y^*_N$. On the other hand, when $M \geq 1$, the latter integrand is larger (since $1 - F(s - \alpha c) \geq 1 - F(s)$), and the latter integral is taken over a larger range (since y^* decreases with N), so $U_{shop} \geq U_{noshop}$, with strict inequality for $y < y^*_{N-M}$. □

Theorem 3.b *Simple bid shopping without diversion increases the procurer's surplus, and reduces the price paid by the procurer (in the first-order stochastic dominance sense).*

Proof. As noted above in Proposition 7, general contractors bid lower for every realization of their lowest pre-auction subcontractor's cost. Thus, the price paid by the procurer is stochastically lower, and the procurer is better off. □

Bid shopping with diversion can either increase or decrease the likelihood of procurement – informally, it will tend to increase it when c is large, but decrease it when c is small. More precisely, the likelihood of procurement is decreasing in N – and thus increases when a pre-auction subcontractor gets diverted to post-auction competition – when $\frac{f(y^*)}{1-F(y^*)} > \frac{1}{r-y^*}$, which is more prone to hold when c is large and therefore y^* is low. The effect on procurer surplus is harder to measure, however, and we don't have a formal result.

As for the effect of bid shopping on prime contractors, even without diversion, the effect is ambiguous:

Theorem 3.c *Simple bid shopping without diversion can increase or decrease a prime contractor's expected surplus, depending on the environment. For example:*

1. *Simple bid shopping* increases *each prime contractor's expected surplus if c is sufficiently large, or if r is sufficiently low*

2. *Simple bid shopping* decreases *each prime contractor's expected surplus if r is not too small, c is sufficiently small, and either (i) post-auction subcontractors have sufficiently high costs, or (ii) $F = F_0 = U[0,1]$ and $n_i \geq 2$ for each prime contractor*

The proof is in the appendix. For an illustrative example, let F and F_0 both be the uniform distribution on $[0,1]$, $\alpha = 0$, $r = 1$, and suppose there are three general contractors, each with three pre-auction subcontractors. When $c = 0.002$, bid shopping without diversion reduces each GC's expected surplus from 0.0513 to 0.0501; when $c = 0.075$, bid shopping increases each GC's expected surplus from 0.1003 to 0.1015.

Finally, we already considered the effect of bid shopping on the subcontractors who continue to bid before the auction, but we can also consider the effect on the subcontractor who chooses to wait and compete after the auction instead of bidding before.

Theorem 3.d *Suppose only a single subcontractor is available to bid after the auction, so he will be approached by whichever prime contractor wins.*

1. *The subcontractor bidding after the auction earns lower expected surplus than each subcontractor who bids before the auction.*

2. *If $\alpha > 0$ and c is sufficiently small, then he earns lower expected surplus by bidding after the auction than he would have by bidding before the auction.*

6 Extensions to the "Simple" Model

The last two sections considered a benchmark model of post-auction bid shopping which relied on several key simplifying assumptions:

1. Only one new subcontractor will be approached after the auction

2. Bid shopping does not impose any additional costs on either the winning prime contractor or her incumbent subcontractor

3. The winning prime contractor will honor her prime auction bid, whether or not she reduces her costs through bid shopping

We also focused on a particular symmetric equilibrium of the game with bid shopping. In this section, we will explore the multiplicity question and defend our selection, and relax each of the assumptions above. To build intuition, we will first explore the role of the participation threshold, which is key to understanding many of these results.

For each of the extensions, for simplicity, we focus on the case where bid shopping does not change the number of subcontractors available to bid before the auction (bid shopping without diversion); each result could then be combined with the effect of a reduction in N to understand the effect of bid shopping with diversion, which will depend on whether a reduction in N increases or decreases total surplus.

6.1 Entry and Total Surplus With Simple Bid Shopping

In the benchmark model without bid shopping, we saw that entry was inefficiently low when $r < v$, and exactly at the efficient level when $r = v$.[54] Thus, if $r = v$, a small "exogenous change" in the entry threshold y^* would have only a second-order effect on total surplus.

We also saw that the introduction of simple bid shopping does not change the entry threshold, and therefore does not change the level of pre-auction participation. With bid shopping, however, this level of entry turns out to no longer be efficient: even when $r = v$, entry by subcontractors is now strictly below the welfare-maximizing level when bid shopping is possible. We show this formally in the proof of Proposition 9 below, in the Appendix; but the economic intuition is straightforward and worth exploring.

[54]Since payments – from procurer to prime contractor or prime contractor to subcontractor – are welfare-neutral transfers, the only effect of the reserve price r on total surplus is through its effect on entry. Hence, "welfare is increasing in r when $r < v$" and "welfare is maximized at $r = v$" (restatements of Proposition 3) are equivalent to "entry is below the efficient level when $r < v$" and "entry is efficient at $r = v$," respectively.

In the case without bid shopping, consider the externality caused when the "marginal entrant" (a subcontractor with costs $y = y^*$) enters. If he enters, he'll bid r, so he will only win if no other subcontractor enters. When that happens, the winning prime contractor will earn zero profit, and the procurer will earn surplus $v - r$; both would have earned zero in his absence. So the externality caused by the marginal subcontractor choosing to enter is proportional to $v - r$. When $r < v$, this is positive, so entry is below the efficient level; when $r = v$, this is zero, so entry is efficient.

With bid shopping, however, when the marginal entrant enters and bids $b(y^*)$, the effect is different. If no other subcontractor bids, this bid again causes the prime contractor to bid r and win, and earn 0 profit on average; and again causes the procurer to earn surplus of $v - r$. But it also creates strictly positive expected surplus for the post-auction subcontractor. Thus, even when $r = v$, the marginal entrant imposes a positive externality; so even when $r = v$, entry is strictly below the welfare-maximizing level when there is bid shopping.[55]

Thus, as long as $r \leq v$, with bid shopping, we should expect a marginal increase in the entry threshold to increase total surplus, and a decrease in the entry threshold to decrease it, in a "first-order" sense. The welfare impact of the next few extensions largely follow from this effect.

6.2 Other Symmetric Equilibria with Simple Bid Shopping

The results in the previous section are all based on the symmetric equilibrium with simple bid shopping where the entry threshold y^* is the same as without bid shopping. In that equilibrium, when a subcontractor has costs just below y^*, he "inflates" his bid to be substantially above r, knowing that anticipating bid shopping, the prime contractor will still calculate her expected cost to be below r and bid. This depends, of course, on the prime contractor's beliefs about the cost of the subcontractor who submitted the bid (and her beliefs about the beliefs of the post-auction subcontractor, whose willingness to enter she depends on to lower her cost). This suggests that by changing beliefs, we might be able to alter the equilibrium.

Consider a different possible equilibrium. This time, prime contractors ignore subcontractor bids above r. Specifically, they believe that any bid above r must come from a subcontractor with actual costs $y \geq r$, and therefore that with simple bid shopping, there is no hope of reducing costs below r; expected costs are then above the reserve price, and the prime contractor will rationally not bid.[56]

If subcontractors have no reason to enter unless they will bid r or less, there is again a symmetric equilibrium, but the participation threshold y_* must now satisfy

$$(r - y_*)(1 - F(y_*))^{N-1}(1 - F_0(y_* - \alpha c)) \quad = \quad c$$

since the marginal entrant cannot bid more than r, and will only win when the other $N - 1$ pre-auction subcontractors and the post-auction subcontractor all choose not to enter. By inspection, $y_* < y^*$. Assuming a symmetric, monotone equilibrium, we can again use the Envelope Theorem to

[55] In one of the "worse" symmetric equilibria considered in the next subsection, the marginal entrant bids $b_{\hat{y}}(\hat{y})$ and induces expected costs strictly below r, and therefore generates strictly positive profits for the prime contractor as well; so the net externality is still positive, and entry still below the efficient level.

[56] Off-equilibrium-path bids could also be deterred by beliefs that they must come from a very *low*-cost sub: this would deter entry by post-auction subcontractors, preventing the winning prime contractor from reducing her cost.

calculate subcontractors' expected surplus at each cost level, use it to infer their bid function, and verify this is an equilibrium. We will see below that total surplus with bid shopping is increasing in the entry threshold; so this new equilibrium will have lower total surplus than the "good" one considered earlier.

Not only are there symmetric equilibria with entry thresholds y_* and y^*; there is a continuum of symmetric equilibria with every threshold in between. Recall that $[\underline{y}, \overline{y}]$ is the support of F.

Proposition 9 *Suppose that either (i) $\overline{y} \geq r$ or (ii) the support of F_0 does not extend below $\underline{y} - \alpha c$. With simple bid shopping, there is a continuum of symmetric equilibria, characterized by entry thresholds $\hat{y} \in [y_*, y^*]$. Among these equilibria, total surplus is increasing in the entry threshold. In some environments, all of these equilibria give total surplus higher than the equilibrium without bid shopping; in some environments, some of them do not.*

For an example to illustrate that total surplus in the worse equilibria with bid shopping can be either higher or lower than without bid shopping, let $F = F_0$ be the uniform distribution on $[0, 1]$ and $\alpha = 0$, and let $v = r = 1$. Table 3 shows total surplus without bid shopping, with bid shopping in the "best" symmetric equilibrium, and with bid shopping in the "worst" symmetric equilibrium.

Table 3: Total surplus in the "bad" bid shopping equilibrium

number of subcontractors N	5	5	10	10
bid preparation cost c	0.01	0.25	0.01	0.25
total surplus, no bid shopping	0.800	0.373	0.866	0.388
total surplus, bid shopping, "best" equilibrium y^*	0.821	0.378	0.873	0.389
total surplus, bid shopping, "worst" equilibrium y_*	0.818	0.371	0.872	0.387

In this example, bid shopping is beneficial even in the worst symmetric equilibrium when c is low, while bid shopping introduces some symmetric equilibria with lower total surplus than no bid shopping when c is sufficiently high.[57]

We have chosen to focus on results based on the "best" equilibrium with bid shopping, in part because these other equilibria strike us as less realistic. Consider the problem from the point of view of a prime contractor facing an off-equilibrium-path bid very slightly higher than the highest anticipated equilibrium-path bid $\hat{b}$. To "deter" such a deviation, her equilibrium strategy must be to ignore it, which requires her to believe her expected cost would be above r if that was the only bid she received. To believe that, she must either believe such a deviation came from a fairly high-cost subcontractor (so that even with bid shopping her expected cost is still too high to bid), or believe it came from a fairly low-cost subcontractor (so that post-auction subcontractors won't want to enter and she won't be able to lower her cost). But high-cost subcontractors would not be able to cover their entry cost even if their deviation led to a bid; and low-cost subcontractors earn high enough equilibrium profits to make such a deviation unprofitable even if it worked. If the prime contractor's

[57]For N up to 10,000, total surplus in the "worst" symmetric equilibrium with bid shopping is higher than without bid shopping when $c \leq 0.16$, and lower than without bid shopping when $c \geq 0.18$. Details are in the appendix.

beliefs are restricted to the types of subcontractors who could actually benefit from such a deviation, she should likely want to bid; in which case this would be a profitable deviation for certain types.

We can formalize this intuition by appealing to the Never a Weak Best Response refinement of Kohlberg and Mertens (1986):

Proposition 10 *Suppose that the support of F_0 does not extend below $\underline{y} - \alpha c$. Then the only symmetric equilibrium satisfying the NWBR refinement is the one with entry cutoff y^*.*

Our game is not a standard signaling game, as it has multiple senders (every pre-auction subcontractor) and multiple receivers (the general contractor receiving the pre-auction bid, and the subcontractor she will bid shop with after the auction), so application of refinements requires some care. The analysis is further complicated by the fact that senders of different types do not agree on the ranking of joint mixed best responses of the receivers, a monotonicity property that is crucial for the equivalence of many standard refinements (Cho and Sobel, 1990).

Formally, the NWBR refinement says that at any off-equilibrium-path event (here, a pre-auction bid above the anticipated range), *if* there are subsequent beliefs that would rationalize this as a weak best-response for any type of player, then the off-path beliefs must put all their weight on such types. In this case, for symmetric equilibria with entry thresholds $\hat{y} < y^*$, there exist off-path bids b for which some beliefs would make b a weak best-response for subcontractors with cost level $\hat{y}$, but would not make b a strictly profitable deviation for anyone; and there are no beliefs which would make b a weak best-response for subcontractors with any other cost level without making b strictly profitable for someone. As a result, any equilibrium satisfying NWBR would have to assign beliefs $y = \hat{y}$ in response to the deviation b; but these beliefs would make b a strictly profitable deviation for subcontractors with costs close to $\hat{y}$. On the other hand, in the y^* equilibrium, no such problem exists, as the marginal type y^* cannot raise his bid and still induce a bid from the prime contractor.

6.3 Bid Shopping with Added Costs

The results in the previous section followed, in part, from the fact that without diversion, simple bid shopping does not change the entry threshold for pre-auction subcontractors, y^*. This occurred because, *conditional* on the winning prime contractor switching subcontractors after the auction, the price she pays is exactly y_{min}, the actual cost of the incumbent subcontractor she is replacing. This led to the marginal subcontractor's inflation of his bid (padding his profits when he wins to counteract the times he is replaced post-auction) and the prime contractor's deflation of that bid (to account for her anticipated post-auction cost reduction) exactly counteracting each other. This, however, is a knife-edge result; lots of things could break it and cause the entry threshold to change.

One example is any added costs associated with bid shopping. In our baseline model above, we assumed bid shopping imposed no additional costs. Now suppose instead that in addition to having sunk his participation costs, a subcontractor who submits the low pre-auction bid but gets replaced via bid shopping after the auction incurs an additional loss of $\varepsilon > 0$. (Perhaps due to capacity constraints, the subcontractor cannot commit to another job until learning whether he'll be awarded this one; bid shopping delays learning whether his pre-auction sub-bid will be honored, imposing an added opportunity cost.)

If a pre-auction subcontractor with costs y bids b, this induces an expected cost

$$B = (1 - F_0(y - \alpha c))b + F_0(y - \alpha c)y$$

for the prime contractor as before, but an expected payoff of

$$(1 - F_0(y - \alpha c))(b - y) + F_0(y - \alpha c)(-\varepsilon) = B - y - F_0(y - \alpha c)\varepsilon$$

for the subcontractor in the event his is the lowest pre-auction subcontractor bid. That is, to induce an expected cost of B for the prime contractor, the subcontractor must now accept an expected payoff from "winning" of $B - y - F_0(y - \alpha c)\varepsilon$, rather than $B - y$ in the absence of this cost. Since the "threshold type" must still induce the prime contractor to bid the reserve price, the entry threshold y^{**} now must satisfy

$$(r - y^{**} - F_0(y^{**} - \alpha c)\varepsilon)(1 - F(y^{**}))^{N-1} = c$$

so the entry threshold y^{**} must be lower than without bid shopping. The same happens if the added cost is borne by the winning prime contractor (see the appendix for details). As discussed above, total surplus is increasing in the entry threshold, which leads to:

Proposition 11 *"Simple" bid shopping with added costs (incurred by either the winning prime contractor or the incumbent subcontractor) reduces the probability of procurement.*

Depending on the magnitude of the new costs, bid shopping may increase or decrease total surplus relative to no bid shopping; if the former, total surplus increases by less than the expected profit of the post-auction subcontractors.

We can also consider the impact of "costly bid shopping" on subcontractor and procurer surplus. Without bid shopping, a pre-auction subcontractor with cost level $y < y^*$ earned expected surplus

$$U(y) = \int_y^{y^*} (1 - F(s))^{N-1} ds$$

With costless simple bid shopping, this fell to

$$U(y) = \int_y^{y^*} (1 - F(s))^{N-1}(1 - F_0(s - \alpha c))ds$$

With simple bid shopping with added costs (whether incurred by subcontractors or the prime contractor), this falls further

$$U(y) = \int_y^{y^{**}} (1 - F(s))^{N-1}(1 - F_0(s - \alpha c))ds$$

for $y < y^{**}$, and 0 for $y \in [y^{**}, y^*)$. Since $y^{**} < y^*$, subcontractors at every cost level are worse off than without bid shopping, and worse off than with costless bid shopping.

Without added costs, simple bid shopping increased procurer surplus, by reducing prime contractors' equilibrium bids at each level of subcontractor costs. With costly bid shopping, the effect is less clear. In the absence of bid shopping, a prime contractor whose lowest-cost subcontractor has cost $y \leq y^*$ submits an equilibrium bid of

$$y + \frac{c + \int_y^{y^*} (1 - F(s))^{N-1} ds}{(1 - F(y))^{N-1}} \tag{9}$$

since this is the lowest sub-bid she receives. With costly bid shopping, when the costs are borne by the incumbent subcontractor, the same prime contractor will not receive a sub-bid, and therefore not bid, if $y > y^{**}$; if $y \leq y^{**}$, the prime contractor's bid will be[58]

$$y + \frac{c + \int_y^{y^{**}} (1 - F(s))^{N-1} (1 - F_0(s - \alpha c)) ds}{(1 - F(y))^{N-1}} + \varepsilon F_0(y - \alpha c) \tag{10}$$

Since (9) is equal to r when $y = y^*$ and (10) is equal to r when $y = y^{**} < y^*$, a prime contractor's bid is higher (or she does not bid) if her lowest-cost subcontractor has cost close to y^{**}. On the other hand, for cost levels well below y^{**}, the prime contractor's bid may be lower with costly bid shopping than without. If ε is sufficiently small, bid shopping will still raise the procurer's expected surplus, although it will reduce the likelihood of procurement (and the likelihood of paying a price lower than the reserve); if ε is sufficiently large, bid shopping is likely to reduce surplus for everyone but the post-auction subcontractors.

6.4 "More Aggressive" Bid Shopping

So while simple bid shopping itself does not reduce participation by pre-auction subcontractors, any added costs associated with bid shopping do reduce participation. Next, however, we will show that a "more aggressive" form of bid shopping actually *increases* pre-auction participation, further increasing total surplus.

Assume there are no costs to bid shopping, and instead of approaching just one post-auction subcontractor, the winning prime contractor will approach $K \geq 2$ new subcontractors after the auction. For simplicity, consider the limit $\alpha \to 0$, so if all K post-auction subcontractors have costs above the incumbent's cost level y_{min}, none of them enter and the price stays the same at $b(y_{min})$; if one has costs below y_{min}, he enters and the incumbent competes the price down to y_{min}; and if two or more new subcontractors have costs below y_{min}, all those with costs below y_{min} enter, and competition pushes the price paid by the prime contractor down to the second-lowest of their costs. The prime contractor's expected cost, given a pre-auction bid of b from a subcontractor with cost

[58] The subcontractor would bid $b(y) = y + \frac{c + \int_y^{y^{**}} (1-F(s))^{N-1}(1-F_0(s-\alpha c))ds}{(1-F(y))^{N-1}(1-F_0(y-\alpha c))} + \varepsilon \frac{F_0(y-\alpha c)}{1-F_0(y-\alpha c)}$ and the prime contractor's bid would be her expected cost $(1 - F_0(y - \alpha c))b(y) + F_0(y - \alpha c)y$ – see calculations in the appendix.

$y_{min} = y$, will therefore be

$$\begin{aligned} B &= (1-F_0(y))^K b + K(1-F_0(y))^{K-1} F_0(y) y \\ &\quad + \int_0^y x \, d\left[1-(1-F_0(x))^K - K(1-F_0(x))^{K-1}F_0(x)\right] \end{aligned}$$

since $1-(1-F_0(x))^K - K(1-F_0(x))^{K-1}F_0(x)$ is the CDF of the second-lowest cost among the K post-auction subcontractors.[59] Integrating by parts and simplifying gives

$$B = (1-F_0(y))^K b + \left(1-(1-F_0(y))^K\right) y - \int_0^y \left(1-(1-F_0(x))^K - K(1-F_0(x))^{K-1}F_0(x)\right) dx$$

Let $A(y) \equiv \int_0^y \left(1-(1-F_0(x))^K - K(1-F_0(x))^{K-1}F_0(x)\right) dx$, and note that this is positive (since the integrand is the probability two or more post-auction subcontractors have costs below x, and therefore non-negative). We can rearrange and find

$$B - y + A(y) = (1-F_0(y))^K (b-y)$$

and note that the right-hand side is the incumbent subcontractor's post-auction expected payoff. Thus, by bidding enough to give the prime contractor expected costs of B, a pre-auction subcontractor with cost y can now earn an expected payoff of $B - y + A(y)$, contrasted with $B - y$ under simple (costless) bid shopping. Since the marginal entrant induces a general contractor bid of r, the entry threshold must now satisfy

$$(r - y^{**} + A(y^{**}))\,(1-F(y^{**}))^{N-1} = c$$

so the entry threshold is now *higher* than without bid shopping. As a result:

Proposition 12 *With "more aggressive" bid shopping as described above, the project is* more likely *to be successfully procured than without bid shopping; and bid shopping increases total surplus by* more than *the expected profit of the subcontractors bidding after the prime auction.*

It's at first surprising that relative to the simple case analyzed earlier, the threat of *more competitive* post-auction bid shopping induces *greater* pre-auction entry. The reason for this is that rather than paying exactly y whenever she replaces the incumbent subcontractor with a new one, the winning prime contractor now gets an added cost reduction (to costs below y) in some states of the world; and this occurs *in states of the world where the incumbent wasn't going to receive the contract anyway.* The prime contractor therefore receives an added cost reduction that the incumbent subcontractor "doesn't pay for"; this allows pre-auction subcontractors to inflate their bids further and still induce the prime contractor to bid, allowing subcontractors to enter at a higher cost level.

This shows it's possible for bid shopping to *increase* the probability of procurement. This *increases* the combined surplus of the pre-auction players (procurer, prime contractors, and pre-auction

[59] Since the subcontractors willing to enter are all those with costs below y_{min}, the second-lowest cost among those who enter is simply the second-lowest cost among those approached.

subcontractors) above what it was with simple bid shopping (or without bid shopping), and increases total surplus beyond what it would be with simple bid shopping (at least when $\alpha = 0$).

It's not immediately obvious, however, to whom this added surplus accrues. Recall that without bid shopping, a pre-auction subcontractor with costs $y < y^*$ got expected surplus

$$\int_y^{y^*} (1 - F(s))^{N-1} ds$$

With "more aggressive" bid shopping, a pre-auction subcontractor with costs $y < y^{**}$ now earns

$$\int_y^{y^{**}} (1 - F(s))^{N-1} (1 - F_0(s))^K ds$$

Since $y^{**} > y^*$ now, it's not obvious whether this is higher or lower than without bid shopping – the integrand is smaller at each point, but the integral is taken over a wider range. For $y \in [y^*, y^{**})$ (and for y just below y^* by continuity), the subcontractor is strictly better off with bid shopping; for y lower than that, it's unclear whether the subcontractor is better or worse off.

Similarly, we saw earlier that for every cost level $y \leq y^*$ of a pre-auction subcontractor, simple bid shopping led to a lower equilibrium bid from the prime contractor, and so bid shopping increased procurer surplus at each realization of subcontractor costs, and did not change the probability of procurement. With more aggressive bid shopping, at subcontractor cost levels between y^* and y^{**}, prime contractors now bid below r instead of not bidding, so the procurer is more likely to buy the project, and to pay less than r, than without bid shopping. At subcontractor cost levels well below y^*, however, it's not clear whether prime contractor bids will be higher or lower than without bid shopping, and therefore whether procurer surplus will go up or down for those realized cost levels.[60] That said, in an environment with c sufficiently large (and y^* therefore close to $\underline{y}$), it's likely that this more aggressive form of bid shopping would represent a Pareto improvement ex ante, improving expected payoffs for subcontractors, prime contractors, and the procurer relative to no bid shopping.

6.5 Ex Post Regret by the Prime Auction Winner

One other concern with bid shopping is that it introduces the possibility of ex post regret on the part of the winning prime contractor. Expecting to bid shop if she wins, a prime contractor (rationally) bids her expected cost, which is less than the lowest sub-bid she's received so far. If she narrowly wins the prime auction, then fails to improve her cost through bid shopping, the payment from the procurer will be less than her cost to complete the project – ex post, she will be facing a loss.

If contracts are ironclad, this isn't a problem – the prime contractor maximized her expected surplus, and has to live with an unlucky draw of ex post profits. (If prime contractors are risk averse, this risk would raise their bids some, but wouldn't fundamentally change anything.) However, in some settings we might worry that prime contractors facing a loss would attempt to get out of the

[60] More specifically, switching from no bid shopping to "more aggressive" bid shopping changes the price paid by the procurer from $y + \frac{c + \int_{\underline{y}}^{y^*} (1-F(s))^{N-1} ds}{(1-F(y))^{N-1}}$ to $y + \frac{c + \int_{\underline{y}}^{y^{**}} (1-F(s))^{N-1} (1-F_0(s))^K ds}{(1-F(y))^{N-1}} - A(y)$ when the second-lowest-cost general contractor's lowest-cost subcontractor has cost level y; for $y < y^*$, it's ambiguous which is greater.

contract – either to walk away entirely, or to renegotiate terms with the procurer. Since this could impose risk, delays, or added costs on the procurer, it's something the procurer might reasonably worry about.

Exploring this topic thoroughly would require a different model, but our model can show that two obvious potential responses to this concern would likely backfire, or at least undermine much of the possible benefit of bid shopping. Consider our baseline model of bid shopping – "simple" bid shopping with no added costs – and consider two candidate rules for the prime auction:

1. Bid shopping is allowed, but prime contractors may only bid if they've received a subcontractor bid of r or less; if they have, they can bid however they want.

2. Bid shopping is allowed, but prime contractors cannot bid less than the lowest sub-bid they've received before the auction.

Our result is that both of these rules lower pre-auction participation down to the level of the "worst" symmetric equilibrium considered earlier. Let the "y_* equilibrium" refer to the symmetric equilibrium with the lowest entry threshold in the simple bid shopping game described earlier, which therefore (per Proposition 9) has the lowest total surplus of the symmetric equilibria.

Proposition 13 *Under simple bid shopping with either of the two rules described above, there is a unique symmetric equilibrium, which has pre-auction entry threshold y_*.*

1. *Under Rule 1, equilibrium payoffs are those of the y_* equilibrium.*

2. *Under Rule 2, total surplus is the same as in the y_* equilibrium; equilibrium payoffs are the same as in the y_* equilibrium* except *for a transfer of surplus from the procurer to the prime contractors.*

Combining this with Proposition 9, then, total surplus under either Rule 1 or Rule 2 is lower than in the absence of these rules, and could be higher or lower than total surplus without bid shopping, depending on the environment.

The intuition for Proposition 13 is as follows. Rule 1 effectively rules out pre-auction subcontractor bids above r, since they won't induce the prime contractor to bid. This has the same effect as deterring bids above r via off-equilibrium-path beliefs, and therefore effectively forces the players into the worst symmetric equilibrium of the simple-bid-shopping game, in which sub-bids above r were ignored.

As for Rule 2, relative to Rule 1, it does not change subcontractor bidding at all – subcontractors still enter with costs $y \leq y_*$, and bid exactly as before, since this is required by incentive compatibility and the Envelope Theorem. All that changes is that prime contractors bid their lowest-yet-received bid, rather than their expected cost, and therefore receive an extra windfall profit in the event bid shopping is successful. Thus, Rule 2 would *cause* an outcome decried in the U.S. Senate report in 1955, but seemingly at odds with equilibrium play – that subcontractors would inflate their bids in response to bid shopping, but that the procurer would not get the benefit of the lower prices realized ex post.[61]

Overall, then, relative to ordinary bid shopping, Rule 1 selects the "worst" symmetric equilibrium (the one with the lowest total surplus, which may be higher or lower than total surplus without bid shopping); and relative to Rule 1, Rule 2 leaves total surplus unchanged, just shifts surplus from the procurer to the winning prime contractor. If breach or post-auction renegotiation is a concern with bid shopping, then, rules of this sort seem a poor way to handle it.

7 Conclusion

The possibility of bid shopping can affect private and public procurement in several ways. In anticipation of being bid shopped, subcontractors will alter their bids to the general contractors. Equally, anticipating their ability to bid shop after winning a procurement auction, general contractors will modify their pre-award bids to the procurer. Finally, bid shopping can affect the participation rate, the threshold level of the job cost above which a subcontractor no longer finds it worthwhile to incur the up-front cost to prepare a bid.

One of the surprising results of our paper is that bid shopping that does not divert subcontractors from the pre-auction competition does *not* alter the participation level. This is because conditional on winning the auction, the marginal subcontractor's expected loss from being bid shopped is exactly offset by the general contractor's expected gain. This invariance of the participation threshold has two immediate consequences: (1) bid shopping does not affect the likelihood with which the project will be implemented; and (2) whenever bid shopping is successful, the project is carried out at lower cost, so bid shopping enhances efficiency. In fact, since the post-auction subcontractor earns exactly the difference between his cost and that of the winning pre-auction subcontractor, the expected gain in surplus from bid shopping equals the expected profit of the post auction subcontractor.

This efficiency gain from bid shopping without diversion should not be interpreted as an endorsement of the practice of bid shopping. Rather than introducing new subcontractors into the competition, bid shopping might well instead divert some existing subcontractors from pre- to post-auction competition. Our results indicate that in this case, anti-bid-shopping legislation would likely enhance welfare if bid preparation costs are low, but could decrease welfare otherwise.

If bid preparation costs in federal contracting are a substantial fraction of the subcontractor's cost of completing a project, a condition that seems quite likely to hold in practice,[62] this may explain why the federal government has steadfastly resisted attempts to curb bid shopping over a period that spans almost a century. Another potential motive – that the government is more concerned about obtaining a fair price for the taxpayer than in enhancing social welfare– is also supported by the predictions of our model, as in the absence of diversion, the procuring agency receives lower bids

[61] "Present bidding procedures cause the subcontractor submitting a bid "to bid so high that he, the subcontractor, can still come down and get the job." ...As long as subcontractors will not submit their final price prior to the award of the prime contract because of bid shopping after the award, the Government cannot get the full benefit of the low competitive price" (United States Senate Committee on the Judiciary, 1955, p. 8).

[62] As projects become more complex, the onus of proving that a company is capable of delivering the subcontracted work becomes more substantial, sometimes even including the cost of building a mock-up (see General Dynamics, ASBCA Nos. 15394 and 15858).

when post-auction bid shopping is permitted. This is so because even though the expectation of being bid shopped sometimes leads subcontractors to bid higher than in the absence of bid shopping, the prime contractors bid more aggressively due to the prospect of successfully reducing their cost after winning the prime auction.

Our results also shed light on other aspects of the political economy of bid shopping. The strong, vociferous and universal opposition of subcontractor associations to the presence of bid shopping cannot simply be dismissed as ex-post regret by disgruntled subcontractors who found themselves bid shopped. Indeed, a closer reading of the history reveals that these associations are strongly concerned about the ex-ante effects on subcontractors' profits and participation rates. Concerning the former, our findings indicate that bid shopping decreases pre-auction subcontractors' profits if and only if there is no diversion, suggesting that diversion may not be very prevalent.

As emphasized in the Introduction and in Section 3, general contractors pay lip service to subcontractors' concern with bid shopping. Generals do not wish to be viewed as engaging in 'unethical' practices, yet oppose efforts to introduce bid shopping legislation such as mandatory listing requirements. It may be that this ambivalence is due to the ambiguous effect bid shopping has on the expected profits of the prime contractors. If there is substantial variation in the procurer's reserve price and the bid preparation costs of the subcontractors, reaching agreement among the prime contractors on the merits or demerits of bid shopping may be difficult to achieve.

Our work also suggests several fruitful avenues for further research. First, when bid shopping is allowed, it would be interesting to model subcontractors' choice to bid before or after the prime auction, thereby generating a prediction about the equilibrium degree of diversion. Second, for tractability, our model assumes that those subcontractors who submit pre-auction bids do not engage in post-auction bid shopping or bid peddling. A useful alternative would be to consider post-auction bid shopping among pre-auction subcontractors only. Third, our paper compares the regimes where every prime contractor bid shops to one where none of them do. As such, we have little to say about the genesis of bid shopping. For example, it might be that if no one bid shops, it is beneficial for any general contractor to bid shop, yet if all of them do all of them are worse off. That is, the phenomenon of bid shopping might very well be a prisoner's dilemma from the general contractors' point of view. Fourth, and perhaps most importantly, our model has assumed that the number of subcontractors remains the same whether or not there is bid shopping. Yet as bid shopping affects the profitability of subcontractors, it may affect their viability or fortune, and hence affect the number of subcontractors that will be present in the market. Some of these extensions may prove quite challenging, for example by breaking the symmetry in the model. We leave these exciting topics for future research.

APPENDIX A – proofs and calculations not in text

Results with short or elegant proofs were mostly proved in the text; proofs that require longer, mostly mechanical calculations are here.

A.1 Proof of last part of Proposition 1 (uniqueness of symmetric equilibrium in absence of bid shopping)

The claim is that the equilibrium in Proposition 1 is the only symmetric equilibrium of the game without bid shopping. The result follows from the following facts:

1. Any equilibrium must use weakly monotone strategies – usual reason that a subcontractor's expected payoff has strictly decreasing differences in his cost and his probability of winning
2. No symmetric equilibrium bid function can be flat over a range of cost types – usual reason that either the highest- or lowest-cost types would benefit from an infinitesimal deviation either upward or downward, respectively
3. Any symmetric equilibrium bid function must have an entry threshold that excludes some bidders – otherwise, given the results above, the highest-cost subcontractors would never win but would still incur the entry cost
4. Any symmetric equilibrium bid function β must have range $[\underline{b}, r]$ for some $\underline{b}$ – i.e., it must go all the way up to r (otherwise subcontractor types bidding the highest equilibrium bid would benefit from bidding r instead), and it must not have holes (otherwise bidders bidding "just below" a gap in the range should deviate to the top of the gap)
5. Any symmetric equilibrium must have the entry threshold y^* in Proposition 1 – if not, then either marginal entrants are strictly profitable (in which case some excluded types could deviate and bid r for positive surplus), or strictly unprofitable (in which case some included types earn negative payoff)
6. Any symmetric equilibrium must therefore use the bid function in Proposition 1 – it follows from the Envelope Theorem, which is equivalent to incentive-compatibility and the equilibrium allocation rule (which is known for any symmetric monotone equilibrium given its entry threshold)

A.2 Proof of Proposition 2 (surplus of PC 1 is increasing in n_1)

Suppose that prime contractor 1 (PC1) solicits one more sub-bid, increasing n_1, leaving $n_{-1} = \sum_{i \neq 1} n_i$ unchanged, and increasing $N = n_1 + n_{-1}$. We claim this increases PC1's expected surplus.

Note first that y^* goes down, because the left-hand side of $(r - y^*)(1 - F(y^*))^{N-1} = c$ is decreasing in both N and y^*, so when N goes up, y^* must go down. This means that the probability that none of PC1's competitors receives a subcontractor bid, $(1 - F(y^*))^{n_{-1}}$, goes up, increasing the likelihood that PC1 gets paid the reserve price if she bids.

Next, note that despite y^* going down, the likelihood that PC1 receives a subcontractor bid goes up. To see this, write the entry threshold condition as

$$\left[(r - y^*)(1 - F(y^*))^{n_{-1}-1}\right](1 - F(y^*))^{n_1} \quad = \quad c$$

Since y^* goes down when n_1 increases but n_{-1} stays the same, the term in square brackets goes up, which means $(1 - F(y^*))^{n_1}$ must go down; this is the probability PC1 receives no subcontractor bids.

What we'd like to do is show that more generally, the distribution of PC1's costs entering the auction (i.e., the distribution of the lowest subcontractor bid received) improves, while the distribution of her opponents' costs gets worse. If we let X_i denote PC i's lowest subcontractor bid and G_i its CDF, then

$$G_i(x) \quad = \quad \Pr(X_i \le x) \quad = \quad 1 - (\Pr(\beta(y_j) \ge x))^{n_i} \quad = \quad 1 - \left(1 - F\left(\beta^{-1}(x)\right)\right)^{n_i}$$

Calculating G_i is messy, but calculating G_i^{-1}, the bid at a given percentile of the distribution, is more feasible. Inverting the right-hand expression shows that $G_i(x) = k$ if and only if

$$x \quad = \quad \beta\left(F^{-1}\left(1 - (1-k)^{\frac{1}{n_i}}\right)\right)$$

and therefore $G^{-1}(k) = \beta\left(F^{-1}\left(1 - (1-k)^{\frac{1}{n_i}}\right)\right)$. Recall that

$$\beta(y) \quad = \quad y + \frac{c + \int_y^{y^*}(1 - F(s))^{N-1}ds}{(1 - F(y))^{N-1}}$$

Writing y as $\int_0^y 1ds$, we can rewrite $\beta(\cdot)$ as

$$\beta(y) \quad = \quad \int_0^{y^*} \min\left\{1, \frac{(1-F(s))^{N-1}}{(1-F(y))^{N-1}}\right\} ds + \frac{c}{(1-F(y))^{N-1}}$$

To calculate $G^{-1}(k) = \beta\left(F^{-1}\left(1 - (1-k)^{\frac{1}{n_i}}\right)\right)$, we evaluate $\beta(y)$ at $y = F^{-1}\left(1 - (1-k)^{\frac{1}{n_i}}\right)$, which means $F(y) = 1 - (1-k)^{\frac{1}{n_i}}$, or $(1 - F(y))^{N-1} = (1-k)^{\frac{N-1}{n_i}}$, and therefore

$$G_i^{-1}(k) \quad = \quad \beta\left(F^{-1}\left(1 - (1-k)^{\frac{1}{n_i}}\right)\right) \quad = \quad \int_0^{y^*} \min\left\{1, \frac{(1-F(s))^{N-1}}{(1-k)^{\frac{N-1}{n_i}}}\right\} ds + \frac{c}{(1-k)^{\frac{N-1}{n_i}}}$$

First, consider the case $i = 1$, so that

$$G_1^{-1}(k) \quad = \quad \int_0^{y^*} \min\left\{1, \frac{(1-F(s))^{N-1}}{(1-k)^{\frac{N-1}{n_1}}}\right\} ds + \frac{c}{(1-k)^{\frac{N-1}{n_1}}}$$

An increase in n_1 decreases $\frac{N-1}{n_1} = 1 + \frac{n_{-1}-1}{n_1}$, and therefore increases $(1-k)^{\frac{N-1}{n_1}}$ since $1 - k < 1$. This increases the denominator of both the integrand and the second term, decreasing both. The increase in N also decreases the integral further, since $(1 - F(s))^{N-1}$ falls for each s. Finally, the decrease in y^* also decreases the first term (without affecting the second). Thus, the overall effect is that $G_1^{-1}(k)$ falls, i.e., each percentile of the distribution of X_1 decreases.

Next, consider $i \neq 1$, so that n_i stays fixed as n_1 (and therefore N) increases. Recalling that $c = (r - y^*)(1 - F(y^*))^{N-1} = \int_{y^*}^r (1 - F(y^*))^{N-1} ds$, rewrite G_i^{-1} as

$$\begin{aligned}
G_i^{-1}(k) \quad &= \quad \int_0^{y^*} \min\left\{1, \tfrac{(1-F(s))^{N-1}}{(1-k)^{\frac{N-1}{n_i}}}\right\} ds + \int_{y^*}^r \tfrac{(1-F(y^*))^{N-1}}{(1-k)^{\frac{N-1}{n_i}}} ds \\
&= \quad \int_0^{y^*} \min\left\{1, \left(\tfrac{1-F(s)}{(1-k)^{\frac{1}{n_i}}}\right)^{N-1}\right\} ds + \int_{y^*}^r \left(\tfrac{1-F(y^*)}{(1-k)^{\frac{1}{n_i}}}\right)^{N-1} ds \\
&= \quad \int_0^r \left(\min\left\{1, \tfrac{1-F(\min\{s,y^*\})}{(1-k)^{\frac{1}{n_i}}}\right\}\right)^{N-1} ds
\end{aligned}$$

With n_i not changing, the increase in $N-1$ increases the integrand (since the term being raised to the $N-1$ power is always at least 1); the decrease in y^* decreases $F(\min\{s, y^*\})$, increasing the integrand as well, and the region of integration does not change, so $G_i^{-1}(k)$ increases for $i \neq 1$.

Putting it all together, then, an increase in n_1 improves the distribution of PC1's costs, and worsens the distribution of her opponents' costs, along with increasing the likelihood she receives a sub-bid and decreasing the likelihood any opponent does, all of which increase her expected profits. $\square$

A.3 Proof of Proposition 4 (the effect of N on total surplus is ambiguous)

Observe that N affects surplus in two ways: directly through its effect on the expression for W and indirectly through its effect on y^*:

$$\frac{dW}{dN} = \frac{\partial W}{\partial N} + \frac{\partial W}{\partial y^*}\frac{\partial y^*}{\partial N}$$

Recall from the proof of Proposition 3 in the text that

$$\frac{\partial W}{\partial y^*} = N(1-F(y^*))^{N-1} f(y^*)(v-r) \tag{11}$$

Totally differentiating (1) (the equation defining y^*) w.r.t. N and solving for $\frac{\partial y^*}{\partial N}$, we obtain

$$\frac{\partial y^*}{\partial N} = -\frac{(r-y^*)(1-F(y^*))[-\ln(1-F(y^*))]}{(1-F(y^*)) + (N-1)(r-y^*)f(y^*)} \tag{12}$$

Combining (11) and (12) gives the total indirect effect:

$$\frac{\partial W}{\partial y^*}\frac{\partial y^*}{\partial N} = -\frac{N(r-y^*)(1-F(y^*))^N f(y^*)(v-r)}{(1-F(y^*)) + (N-1)(r-y^*)f(y^*)}[-\ln(1-F(y^*))] \tag{13}$$

Next, we compute the direct effect. Integrating (3) (the expression for W in the text) by parts gives

$$W = (v-y^*)(1-(1-F(y^*))^N) + \int_{\underline{y}}^{y^*} (1-(1-F(s))^N ds - cNF(y^*)$$

Partially differentiating w.r.t. N and substituting for c from (1) then yields

$$\begin{aligned} \frac{\partial W}{\partial N} = {} & (v-y^*)(1-F(y^*))^N[-\ln(1-F(y^*))] \\ & + \int_{\underline{y}}^{y^*} (1-F(s))^N[-\ln(1-F(s))]ds - (r-y^*)(1-F(y^*))^{N-1}F(y^*) \end{aligned} \tag{14}$$

The highest possible value of c for which the market does not collapse equals $r - \underline{y}$. At this value we have $y^* = \underline{y}$, $F(y^*) = 0$ and $\ln(1-F(y^*)) = 0$, so that both the direct and indirect effect vanish as $c \uparrow (r - \underline{y})$.

To examine the behavior of $\frac{dW}{dN}$ in a left neighborhood of $(r-\underline{y})$, we shall factor out the term $F(y^*)$ in (13) and (14), and consider the limiting behavior of the remaining term. For this purpose, note that by l'Hospital's rule we have $\lim_{x\to 0} \frac{\ln(1-x)}{x} = 1$. We may then compute

$$\begin{aligned}
\lim_{c\uparrow(r-\underline{y})} \frac{1}{F(y^*)}\frac{\partial W}{\partial y^*}\frac{\partial y^*}{\partial N} &= -\lim_{y^*\downarrow \underline{y}} \frac{N(r-y^*)(1-F(y^*))^N f(y^*)(v-r)}{(1-F(y^*))+(N-1)(r-y^*)f(y^*)} \frac{[-\ln(1-F(y^*))]}{F(y^*)} \\
&= -\frac{N(r-\underline{y})f(\underline{y})(v-r)}{1+(N-1)(r-\underline{y})f(\underline{y})}
\end{aligned}$$

and

$$\begin{aligned}
\lim_{c\uparrow(r-\underline{y})} \frac{1}{F(y^*)}\frac{\partial W}{\partial N} &= \lim_{y^*\downarrow \underline{y}} \Bigg\{ (v-y^*)(1-F(y^*))^N \frac{[-\ln(1-F(y^*))]}{F(y^*)} \\
&\quad + \frac{\int_{\underline{y}}^{y^*}(1-F(s))^N[-\ln(1-F(s))]ds}{F(y^*)} - (r-y^*)(1-F(y^*))^{N-1} \Bigg\} \\
&= \lim_{y^*\downarrow \underline{y}} \Bigg\{ (v-y^*)(1-F(y^*))^N + \frac{(1-F(y^*))^N[-\ln(1-F(y^*))]}{f(y^*)} \\
&\quad -(r-y^*)(1-F(y^*))^{N-1} \Bigg\} \\
&= v-r
\end{aligned}$$

Since $F(y^*) = f(\underline{y})(y^*-\underline{y}) + o(y^*-\underline{y})$, we therefore obtain

$$\frac{dW}{dN} = f(\underline{y})(y^*-\underline{y})(v-r)\left[\frac{1-(r-\underline{y})f(\underline{y})}{1+(N-1)(r-\underline{y})f(\underline{y})}\right] + O(y^*-\underline{y})$$

Thus for c sufficiently close to $r-\underline{y}$ aggregate surplus is increasing in N when $f(\underline{y}) > \frac{1}{r-\underline{y}}$.

On the other hand, as $c \to 0$, the left-hand side of (1) must also go to zero, meaning y^* approaches the smaller of $\bar{y}$ and r. This means $(r-y^*)(1-F(y^*)) \to 0$, so $\frac{\partial y^*}{\partial N} \to 0$, and the indirect effect of N (through y^*) therefore vanishes; what remains is

$$\begin{aligned}
\lim_{c\downarrow 0} \frac{dW}{dN} &= \lim_{c\downarrow 0} \frac{\partial W}{\partial N} \\
&= \lim_{c\downarrow 0} \left\{ (v-y^*)(1-F(y^*))^N[-\ln(1-F(y^*))] + \int_{\underline{y}}^{y^*} (1-F(s))^N[-\ln(1-F(s))]ds \right\} \\
&\geq \int_{\underline{y}}^{\min\{r,\bar{y}\}} (1-F(s))^N[-\ln(1-F(s))]ds \qquad > \qquad 0
\end{aligned}$$

so welfare increases with N when c is sufficiently small. □

A.4 Proof of Proposition 5 (equilibrium with "simple" bid shopping)

First, let us show that in any symmetric monotone equilibrium with entry threshold y^* the bid function must be (7). By the envelope theorem, the expected payoff of a pre-auction subcontractor with cost $y < y^*$ must equal

$$U(y) \quad = \quad \int_y^{y^*} (1 - F(s))^{N-1}(1 - F_0(s - \alpha c))ds.$$

A pre-auction subcontractor's expected payoff from being the low pre-auction bid ("winning" the auction) equals $(1 - F_0(y - \alpha c))(b(y) - \alpha c)$, so that

$$(1 - F(y))^{N-1}(1 - F_0(y - \alpha c))(b(y) - y) - c \quad = \quad \int_y^{y^*} (1 - F(s))^{N-1}(1 - F_0(s - \alpha c))ds$$

Solving for $b(y)$ yields (7). Differentiating w.r.t. y, we can verify that this bid function is strictly increasing in y.

Faced with the bid function (7), a general contractor whose lowest sub-bid is from a subcontractor with cost y, bids

$$C(b(y), y) \quad = \quad F_0(y - \alpha c)y + (1 - F_0(y - \alpha c))b(y) \quad = \quad y + \frac{c + \int_y^{y^*} (1 - F(s))^{N-1}(1 - F_0(s - \alpha c))ds}{(1 - F(y))^{N-1}}$$

in the prime auction. Differentiating $C(b(y), y)$ with respect to y, we can verify that this is indeed strictly increasing. Furthermore, if the lowest sub-bid comes from a subcontractor with cost y^*, the general contractor will bid

$$C(b(y^*), y^*) \quad = \quad y^* + \frac{c}{(1 - F(y^*))^{N-1}} \quad = \quad y^* + (r - y^*) \quad = \quad r,$$

justifying the choice of the entry threshold y^*.

It remains to be shown that a subcontractor with cost y cannot profitably deviate by submitting a bid different from $b(y)$.

First, consider a bid $b(y')$ with $y' \leq y + \alpha c$. Then he will win whenever all his pre-auction opponents have costs above y', and the post-auction subcontractor has cost above $y' - \alpha c$ (and therefore doesn't enter), giving expected payoff

$$\begin{aligned} U(y, b(y')) \quad &= \quad (1 - F(y'))^{N-1}(1 - F_0(y' - \alpha c))\,(b(y') - y) \\ &= \quad (1 - F(y'))^{N-1}(1 - F_0(y' - \alpha c))\,(b(y') - y') + (1 - F(y'))^{N-1}(1 - F_0(y' - \alpha c))\,(y' - y) \\ &= \quad c + \int_{y'}^{y^*} (1 - F(s))^{N-1}(1 - F_0(s - \alpha c))ds + (1 - F(y'))^{N-1}(1 - F_0(y' - \alpha c))\,(y' - y) \end{aligned}$$

Differentiating with respect to y' gives

$$\begin{aligned} \frac{d}{dy'}U(y, b(y')) \quad &= \quad -(1 - F(y'))^{N-1}(1 - F_0(y' - \alpha c)) + (1 - F(y'))^{N-1}(1 - F_0(y' - \alpha c)) \\ &\quad + \left(\frac{d}{dy'}\left[(1 - F(y'))^{N-1}(1 - F_0(y' - \alpha c))\right]\right)(y' - y) \end{aligned}$$

The first two terms cancel, and since $(1 - F(y'))^{N-1}(1 - F(y' - \alpha c))$ is decreasing in y', the deviation payoff is proportional to $y - y'$, hence single peaked in y' over the interval $[\underline{y}, y + \alpha c]$, attaining a maximum at $y' = y$.

Next, consider a deviation to a bid $b(y')$ for $y' \in (y + \alpha c, y^*]$. In this case, the deviating sub will sometimes let in a post-auction entrant that he can beat ex-post. Since this will happen whenever the late entrant has cost between y and $y' - \alpha c$, the expected payoff from deviating over this range equals

$$\begin{aligned} U(y, b(y')) &= (1-F(y'))^{N-1}(1-F_0(y'-\alpha c))\,(b(y')-y) + (1-F(y'))^{N-1}\int_y^{y'-\alpha c}(s-y)dF_0(s) \\ &= c + \int_{y'}^{y^*}(1-F(s))^{N-1}(1-F_0(s-\alpha c))ds + (1-F(y'))^{N-1}(1-F_0(y'-\alpha c))\,(y'-y) \\ &+\ (1-F(y'))^{N-1}\int_y^{y'-\alpha c}(s-y)dF_0(s) \end{aligned}$$

Differentiating with respect to y' and then simplifying,

$$\begin{aligned} \frac{dU(y,b(y'))}{dy'} &= -(N-1)(1-F(y'))^{N-2}f(y')(1-F_0(y'-\alpha c))(y'-y) \\ &\quad -(N-1)(1-F(y'))^{N-2}f(y')\int_y^{y'-\alpha c}(s-y)dF_0(s) - (1-F(y'))^{N-1}f_0(y'-\alpha c)(\alpha c) \end{aligned}$$

which is unambiguously negative for $y' \geq y + \alpha c$. So if a sub with cost y wanted to bid at least $b(y+\alpha c)$, he would want to bid exactly $b(y+\alpha c)$, which we already know does worse than $b(y)$.

Any subcontractor bids above $b(y^*)$ are ignored by the general contractor, rationalized by beliefs that that they come from a subcontractor with costs at least y^* and would therefore give expected cost more than r. Any bid below $b(\underline{y})$ is dominated by $b(\underline{y})$, as this equilibrium bid is already revealing the subcontractor to have the lowest cost he could have and thus deterring entry by the maximal set of post-auction subcontractors; no beliefs would lead a post-auction subcontractor to enter less often unless the pre-auction sub bid below $\underline{y}$, which would lead to negative payoffs. □

A.5 Proof of Proposition 6 (effect of bid shopping on subcontractor bids)

Let $\Delta(y) = b(y) - \beta(y)$. It follows from (2) and (7) that

$$\begin{aligned} \Delta(y) &= \frac{c + \int_y^{y^*}(1-F(s))^{N-1}(1-F_0(s-\alpha c))ds}{(1-F(y))^{N-1}(1-F_0(y-\alpha c))} - \frac{c+\int_y^{y^*}(1-F(s))^{N-1}ds}{(1-F(y))^{N-1}} \\ &= \frac{cF_0(y-\alpha c) + \int_y^{y^*}(1-F(s))^{N-1}\left[F_0(y-\alpha c) - F_0(s-\alpha c)\right]ds}{(1-F(y))^{N-1}(1-F_0(y-\alpha c))} \end{aligned}$$

Plugging in $y = y^*$ gives

$$\Delta(y^*) = \frac{cF_0(y^*-\alpha c)}{(1-F(y^*))^{N-1}(1-F_0(y^*-\alpha c))} > 0$$

On the other hand, if $F_0(y-\alpha c) = 0$,

$$\Delta(y) = \frac{-\int_y^{y^*}(1-F(s))^{N-1}F_0(s-\alpha c)ds}{(1-F(y))^{N-1}} < 0$$

For the third result, note that the denominator in the expression for $\Delta(y)$ is strictly decreasing in y. If $\Delta(y) \geq 0$ then $F_0(y - \alpha c) > 0$, in which case the derivative of the numerator with respect to y is

$$f_0(y - \alpha c)\left[c + \int_y^{y^*} (1 - F(s))^{N-1} ds\right] \quad \geq \quad 0$$

and $\Delta(y)$ is therefore strictly increasing. Since $\Delta(y)$ is strictly increasing whenever it is weakly positive, it crosses zero at most once, from below. $\square$

A.6 Proof of Theorem 2.b (welfare effect of bid shopping with diversion)

Let y_N^* denote the entry threshold y^* when there are N pre-auction subcontractors. Plugging in $F_0 = F$ in (8), the combined expected surplus of all post-auction subcontractors, gives

$$\begin{aligned} U_N &= \int_{\underline{y}}^{y_N^*} \int_{\underline{y}}^{y-\alpha c} F(s) N (1 - F(y))^{N-1} f(y) ds dy \\ &= \int_{\underline{y}}^{y_N^* - \alpha c} \int_{s+\alpha c}^{y_N^*} N(1 - F(y))^{N-1} f(y) dy F(s) ds \\ &= \int_{\underline{y}}^{y_N^* - \alpha c} \left[(1 - F(s + \alpha c))^N - (1 - F(y_N^*))^N\right] F(s) ds \end{aligned}$$

From (3), total surplus without bid shopping and with N subcontractors is

$$\begin{aligned} W_{noshop,N} &= \int_{\underline{y}}^{y_N^*} (v - s) d\left(1 - (1 - F(s))^N\right) - NF(y_N^*)c \\ &= (v - y_N^*)\left(1 - (1 - F(y_N^*))^N\right) + \int_{\underline{y}}^{y_N^*} \left(1 - (1 - F(s))^N\right) ds - NF(y_N^*)c \end{aligned}$$

From the proof of Theorem 1, total surplus with bid shopping when there are N pre-auction subcontractors equals

$$W_{shop,N} \quad = \quad W_{noshop,N} + U_N$$

We are interested in the sign of $\Omega \equiv W_{shop,N-M} - W_{noshop,N}$ for small values of c.

First, suppose that $\overline{y} > r$, so that as $c \to 0$ the entry threshold goes to r, regardless of the value of N. Then

$$\begin{aligned} \lim_{c \to 0} \Omega &= (v - r)(1 - (1 - F(r))^{N-M}) + \int_{\underline{y}}^{r} (1 - (1 - F(s))^{N-M}) ds \\ &\quad + \int_{\underline{y}}^{r} \left[(1 - F(s))^{N-M} - (1 - F(r))^{N-M}\right] F(s) ds \\ &\quad - (v - r)\left(1 - (1 - F(r))^N\right) - \int_{\underline{y}}^{r} (1 - (1 - F(s))^N) ds \\ &= -(v - r)\left((1 - F(r))^{N-M} - (1 - F(r))^N\right) - \int_{\underline{y}}^{r} \left[(1 - F(s))^{N-M} - (1 - F(s))^N\right] ds \\ &\quad + \int_{\underline{y}}^{r} \left[(1 - F(s))^{N-M} - (1 - F(r))^{N-M}\right] F(s) ds \\ &= -(v - r)\left((1 - F(r))^{N-M} - (1 - F(r))^N\right) - \int_{\underline{y}}^{r} \left[(1 - F(s))^{N-M}(1 - F(s)) - (1 - F(s))^N\right] ds \\ &\quad - \int_{\underline{y}}^{r} (1 - F(r))^{N-M} F(s) ds, \end{aligned}$$

which is strictly negative for $1 \leq M \leq N$. By continuity, then, $W_{shop,N-M} < W_{noshop,N}$ for $1 \leq M \leq N$ and c sufficiently small.

Next, suppose that $\overline{y} < r$, so that the entry threshold goes to $\overline{y}$ as $c \to 0$. In that case,

$$\begin{aligned}
\lim_{c\to 0}\Omega &= (v-\overline{y})(1-(1-F(\overline{y}))^{N-M}) + \textstyle\int_{\underline{y}}^{\overline{y}}(1-(1-F(s))^{N-M})ds \\
&\quad + \textstyle\int_{\underline{y}}^{\overline{y}}\left[(1-F(s))^{N-M}-(1-F(\overline{y}))^{N-M}\right]F(s)ds - (v-\overline{y})\left(1-(1-F(\overline{y}))^{N}\right) \\
&\quad - \textstyle\int_{\underline{y}}^{\overline{y}}(1-(1-F(s))^{N})ds \\
&= (v-\overline{y}) + \textstyle\int_{\underline{y}}^{\overline{y}}(1-(1-F(s))^{N-M})ds + \textstyle\int_{\underline{y}}^{\overline{y}}(1-F(s))^{N-M}F(s)ds - (v-\overline{y}) \\
&\quad - \textstyle\int_{\underline{y}}^{\overline{y}}(1-(1-F(s))^{N})ds \\
&= \textstyle\int_{\underline{y}}^{\overline{y}}[(1-F(s))^{N}-(1-F(s))^{N-M}(1-F(s))]ds
\end{aligned}$$

For $M > 1$, this is expression is strictly negative. By continuity, for c sufficiently small, bid shopping is welfare-negative when it diverts more than one pre-auction subcontractor.

However, for $M = 1$, this expression equals zero. In that case, we will calculate the sign of Ω by calculating $\frac{d\Omega}{dc}|_{c=0}$. Differentiating $W_{noshop,N}$ yields

$$\frac{d}{dc}W_{noshop,N} = (v-y_N^*)N(1-F(y_N^*))^{N-1}f(y_N^*)\frac{\partial y_N^*}{\partial c} - Nf(y_N^*)c\frac{\partial y_N^*}{\partial c} - NF(y_N^*)$$

Taking limits of this expression requires some care as we claim that $\frac{\partial y_N^*}{\partial c} \to \infty$ as $c \to 0$. Indeed, from (1) we may calculate

$$-(1-F(y_N^*))^{N-2}[1-F(y_N^*)+(N-1)(r-y_N^*)f(y_N^*)]\frac{\partial y_N^*}{\partial c} = 1,$$

Since $y_N^* \to \overline{y}$ as $c \to 0$, it follows that

$$-(1-F(y_N^*))^{N-2}\frac{\partial y_N^*}{\partial c} \to -\frac{1}{(N-1)(r-\overline{y})f(\overline{y})},$$

which is bounded because $r > \overline{y}$. It follows that $(1-F(y_N^*))^{N-1}f(y_N^*)\frac{\partial y_N^*}{\partial c}$ and $c\frac{\partial y_N^*}{\partial c} = (r-y_N^*)(1-F(y_N^*))^{N-1}\frac{\partial y_N^*}{\partial c}$ both converge to zero as c vanishes, and hence that

$$\left.\frac{d}{dc}W_{noshop,N}\right|_{c=0} = -N$$

As a result we may calculate $\left.\frac{d\Omega}{dc}\right|_{c=0} = \left.\frac{d}{dc}\left(W_{shop,N-1} - W_{noshop,N}\right)\right|_{c=0}$ as follows:

$$\begin{aligned}
\left.\tfrac{d\Omega}{dc}\right|_{c=0} &= -(N-1) + N + \left.\tfrac{dU_N}{dc}\right|_{c=0} \\
&= 1 + \lim\nolimits_{c\to 0}\Big\{\left(\tfrac{\partial y_{N-1}^*}{\partial c} - \alpha\right)\left[\left(1-F(y_{N-1}^*)\right)^{N-1} - \left(1-F(y_{N-1}^*)\right)^{N-1}\right]F(y_{N-1}^*) \\
&\quad + \textstyle\int_{\underline{y}}^{y_{N-1}^*-\alpha c}\left[-\alpha(N-1)(1-F(s+\alpha c))^{N-2}f(s+\alpha c)\right]F(s)ds \\
&\quad + \textstyle\int_{\underline{y}}^{y_{N-1}^*-\alpha c}\left[(N-1)(1-F(y_{N-1}^*))^{N-2}f(y_{N-1}^*)\tfrac{\partial y_{N-1}^*}{\partial c}\right]F(s)ds\Big\} \\
&= 1 - \textstyle\int_{\underline{y}}^{\overline{y}}\left[\alpha(N-1)(1-F(s))^{N-2}f(s)\right]F(s)ds \\
&= 1 - \alpha\textstyle\int_{\underline{y}}^{\overline{y}}F(s)d\left(1-(1-F(s))^{N-1}\right) \quad > \quad 0,
\end{aligned}$$

where the third equality follows because as shown above $(1-F(y^*_{N-1}))^{N-2}\frac{\partial y^*_{N-1}}{\partial c} \to 0$, and the ultimate inequality because $1-(1-F(s))^{N-1}$ is a distribution function and $\alpha \leq 1$. Thus when $\overline{y} < r$ then $W_{shop,N-1} - W_{noshop,N} > 0$ for c sufficiently small.

A.7 Proof of Theorem 3.c (effect of bid shopping on PC profits)

Recall that in equilibrium, a prime contractor whose lowest-cost subcontractor has cost $y \leq y^*$, receives a pre-auction bid and bids her expected cost based on that bid, which is $\beta(y)$ without bid shopping, and $(1-F_0(y-\alpha c))b(y) + F_0(y-\alpha c)y$ with simple bid shopping; let $C(y)$ denote either, as appropriate. If the lowest-cost subcontractor of her opponents has cost z, then the lowest competing prime bid (if there is one) is $C(z)$, so she wins the auction when $y < z$, earning a surplus of $C(z) - C(y)$, or $r - C(y)$ if no competitor bids. If we let H be the CDF of the cost level of the lowest competitor's subcontractor, the expected surplus for a prime contractor whose lowest-cost subcontractor has cost y is

$$\begin{aligned}
\Pi(y) &= \textstyle\int_y^{y^*} (C(z)-C(y))dH(z) + (1-H(y^*))(r-C(y)) \\
&= -(C(z)-C(y))(1-H(z))|_{z=y}^{y^*} + \textstyle\int_y^{y^*}(1-H(z))C'(z)dz + (1-H(y^*))(r-C(y)) \\
&= \textstyle\int_y^{y^*}(1-H(z))C'(z)dz
\end{aligned}$$

and, taking the expectation over y, a prime contractor's ex ante expected surplus is

$$\begin{aligned}
\Pi \equiv \textstyle\int_{\underline{y}}^{y^*} \Pi(y)dG(y) &= \int_{\underline{y}}^{y^*}\int_y^{y^*}(1-H(z))C'(z)dzdG(y) \\
&= \textstyle\int_{\underline{y}}^{y^*}\int_{\underline{y}}^{z} dG(z)(1-H(z))C'(z)dz \\
&= \textstyle\int_{\underline{y}}^{y^*} G(z)(1-H(z))C'(z)dz
\end{aligned}$$

where G is the CDF of her own lowest-cost subcontractor's cost. Note $G(y) = 1-(1-F(y))^n$ and $H(z) = 1-(1-F(z))^{N-n}$.

As noted above, the prime contractors's equilibrium bid given y is $\beta(y)$ when there is no bid shopping, and $(1-F_0(y-\alpha c))b(y) + F_0(y-\alpha c)y$ when there is bid shopping, so

$$\begin{aligned}
C_{noshop}(y) &= y + \frac{1}{(1-F(y))^{N-1}}\left(c + \int_y^{y^*}(1-F(s))^{N-1}ds\right) \\
C_{shop}(y) &= y + \frac{1}{(1-F(y))^{N-1}}\left(c + \int_y^{y^*}(1-F(s))^{N-1}(1-F_0(s-\alpha c))ds\right)
\end{aligned}$$

Rather than calculating Π separately with and without bid shopping, it's simpler to calculate the difference directly, as

$$\Delta\Pi \equiv \Pi_{shop} - \Pi_{noshop} = \int_{\underline{y}}^{y^*} G(z)(1-H(z))\left(C'_{shop}(z) - C'_{noshop}(z)\right)dz$$

Defining $\Delta C(\cdot) = C_{shop}(\cdot) - C_{noshop}(\cdot)$, we can calculate

$$\begin{aligned}
\Delta C(y) &= -\frac{1}{(1-F(y))^{N-1}} \left(\int_y^{y^*} (1-F(s))^{N-1} F_0(s-\alpha c) ds \right) \\
\Delta C'(y) &= -\frac{(N-1)}{(1-F(y))^N} f(y) \left(\int_y^{y^*} (1-F(s))^{N-1} F_0(s-\alpha c) ds \right) + F_0(y-\alpha c)
\end{aligned}$$

Plugging this expression for $\Delta C'(\cdot)$ in for $C'_{shop}(z) - C'_{noshop}(z)$ in the expression above for $\Delta\Pi$ (and then splitting the integral into two pieces), we find that for a prime contractor with n pre-auction subcontractors,

$$\begin{aligned}
\Delta\Pi &= -\int_{\underline{y}}^{y^*} \int_z^{y^*} \left(1-(1-F(z))^n\right)(1-F(z))^{N-n} \frac{(N-1)f(z)}{(1-F(z))^N} (1-F(s))^{N-1} F_0(s-\alpha c) ds dz \\
&\quad + \int_{\underline{y}}^{y^*} \left(1-(1-F(z))^n\right)(1-F(z))^{N-n} F_0(z-\alpha c) dz
\end{aligned}$$

Switching the order of integration in the first line, evaluating the inner integral, and simplifying (detailed calculations available upon request), this eventually simplifies to

$$\Delta\Pi = \int_{\underline{y}}^{y^*} (1-F(s))^{N-n} \left[-\frac{N-n}{n-1} \left(1-(1-F(s))^{n-1}\right) + N(1-F(s))^{n-1} F(s) \right] F_0(s-\alpha c) ds$$

Since $1 - x^{n-1} = (1-x)(1+x+x^2+\cdots+x^{n-2})$, plugging in $x = 1-F(s)$ lets us rewrite this as

$$\begin{aligned}
\Delta\Pi &= \int_{\underline{y}}^{y^*} (1-F(s))^{N-n} F(s) \left[-(N-n) \frac{\sum_{j=1}^{n-1} (1-F(s))^{j-1}}{n-1} + N(1-F(s))^{n-1} \right] F_0(s-\alpha c) ds \\
&= \int_{\underline{y}}^{y^*} F(s) F_0(s-\alpha c) \left[-(N-n) \frac{\sum_{j=1}^{n-1} (1-F(s))^{N-1-j}}{n-1} + N(1-F(s))^{N-1} \right] ds
\end{aligned}$$

$\Delta\Pi$ is positive if the integrand is positive everywhere, i.e., if for every $s \in [\underline{y}, y^*]$,

$$\begin{aligned}
(N-n)\text{avg}\left\{(1-F(s))^{N-n}, \cdots, (1-F(s))^{N-2}\right\} &< N(1-F(s))^{N-1} \\
\Leftrightarrow \qquad \text{avg}\left\{(1-F(s))^{-(n-1)}, \cdots, (1-F(s))^{-1}\right\} &< \frac{N}{N-n}
\end{aligned}$$

If $1 - F(y^*) > \left(\frac{N-n}{N}\right)^{\frac{1}{n-1}}$, then $(1-F(y^*))^{-(n-1)} < \frac{N}{N-n}$, in which case $(1-F(s))^{-i} < \frac{N}{N-n}$ for every $s \leq y^*$ and $i \in \{1, 2, \ldots, n-1\}$, which is sufficient for the conditions above to hold and for $\Delta\Pi$ to be positive. Since y^* is determined by

$$(r-y^*)(1-F(y^*))^{N-1} = c$$

having $y^* < x$ is equivalent to $(r-x)(1-F(x))^{N-1} < c$. Plugging in $x = F^{-1}(1 - \left(\frac{N-n}{N}\right)^{\frac{1}{n-1}})$, we need

$$\left(r - F^{-1}\left(1 - \left(\tfrac{N-n}{N}\right)^{\frac{1}{n-1}}\right)\right)\left(\tfrac{N-n}{N}\right)^{\frac{N-1}{n-1}} < c$$

which holds if c is sufficiently large or r sufficiently close to the bottom of the support of F, so in either of these cases, bid shopping increases the expected surplus of the prime contractor in question.

On the other hand, rewriting $\Delta\Pi$ as

$$\Delta\Pi \quad = \quad N \int_{\underline{y}}^{y^*} F(s) F_0(s - \alpha c)(1 - F(s))^{N-1} \left[1 - \frac{N-n}{N} \frac{\sum_{j=1}^{n-1} (\frac{1}{1-F(s)})^j}{n-1} \right] ds,$$

note that as $1 - F(s)$ gets small, the term in square brackets becomes negative. As c falls, y^* increases; if $F_0(s - \alpha c) = 0$ until the term in square brackets is negative, then the integral is negative and bid shopping hurts the prime contractor.

Finally, if we let $F = F_0 = U[0,1]$, then

$$\Delta\Pi \quad = \quad \int_0^{y^*} (1-s)^{N-n} \left[-\frac{N-n}{n-1} \left(1 - (1-s)^{n-1} \right) + N(1-s)^{n-1} s \right] \max\{0, s - \alpha c\} ds$$

If $r \geq 1$ then as $c \to 0$, $y^* \to 1$ (since $(r - y^*)(1 - y^*)^{N-1} = c \to 0$), in which case

$$\begin{aligned} \Delta\Pi \quad &\to \quad \int_0^1 (1-s)^{N-n} \left[-\frac{N-n}{n-1} \left(1 - (1-s)^{n-1} \right) + N(1-s)^{n-1} s \right] s \, ds \\ &= \quad \int_0^1 t^{N-n} \left[-\frac{N-n}{n-1} \left(1 - t^{n-1} \right) + N t^{n-1} (1-t) \right] (1-t) \, dt \end{aligned}$$

with a change of variable to $t = 1 - s$. We can very laboriously calculate this limit integral (step-by-step calculation available by request) to be

$$\begin{aligned} \lim_{c \to 0} \Delta\Pi \quad &= \quad \int_0^1 t^{N-n} (1-t) \left(-\frac{N-n}{n-1} \left(1 - t^{n-1} \right) + N t^{n-1} (1-t) \right) dt \\ &= \quad \frac{1}{n-1} \left[-\frac{N-n}{N-n+1} + \frac{N-n}{N-n+2} + \frac{(N-1)n}{N} - \frac{2Nn-n-N}{N+1} + \frac{N(n-1)}{N+2} \right] \\ &= \quad \frac{1}{n-1} \left[\frac{1}{N-n+1} - \frac{2}{N-n+2} + \frac{N^2+Nn-2n}{N(N+1)(N+2)} \right] \\ &\propto \quad -(N-n-2)(n-1)Nn - 2n(n-2)(n-1) \end{aligned}$$

showing that $\lim_{c \to 0} \Delta\Pi < 0$, and therefore that $\Delta\Pi < 0$ when c is sufficiently small.

A.8 Proof of Theorem 3.d (effect on diverted subcontractor's profits)

For part 1, note that if the post-auction sub has costs $y_0 > y^* - \alpha c$, he gets payoff 0, since whenever he enters someone else has costs less than αc above his own and he won't be able to recover his entry costs. If he has costs $y_0 < y^* - \alpha c$, then any time $y_{min} \in (y_0 + \alpha c, y^*)$, he gets payoff $y_{min} - y_0 - \alpha c$, giving him expected payoff

$$\begin{aligned} U_{late}(y) \quad &= \quad \int_{y+\alpha c}^{y^*} (s - y - \alpha c) d \left(1 - (1 - F(s))^N \right) \\ &= \quad (y^* - y - \alpha c)(1 - (1 - F(y^*))^N) - \int_{y+\alpha c}^{y^*} \left(1 - (1 - F(s))^N \right) ds \\ &= \quad \int_{y+\alpha c}^{y^*} \left[(1 - F(s))^N - (1 - F(y^*))^N \right] ds \end{aligned}$$

As noted in the proof of Theorem 3.a in the text, a pre-auction subcontractor with costs $y < y^*$ earns expected payoff

$$U_{early}(y) \quad = \quad \int_y^{y^*} (1 - F(s))^{N-1}(1 - F_0(s - \alpha c))ds$$

Since $F = F_0$ and $1 - F(s) < 1 - F(s - \alpha c)$, $U_{late}(y) < U_{early}(y)$; since $F = F_0$, the distribution of costs is the same for pre- and post-auction subcontractors, so the expected value of $U_{late}(y)$ is less than the expected value of $U_{early}(y)$.

For part 2, we want to see whether a lone post-auction subcontractor gained by waiting – that is, to compare his payoff from bidding post-auction (in a setting with $N-1$ pre-auction subcontractors) to bidding pre-auction (in a setting with N pre-auction subcontractors and no bid shopping). Letting y^*_{N-1} and y^*_N denote the corresponding entry thresholds, the former expected payoff is

$$\begin{aligned} U_{late} &= E_{y_0 \sim F}\left[\int_{y_0+\alpha c}^{y^*_{N-1}} \left[(1-F(s))^{N-1} - (1-F(y^*_{N-1}))^{N-1}\right] ds\right] \\ &= \int_{\underline{y}}^{y^*_{N-1}-\alpha c} \int_{y_0+\alpha c}^{y^*_{N-1}} \left[(1-F(s))^{N-1} - (1-F(y^*_{N-1}))^{N-1}\right] f(y_0) ds dy_0 \\ &= \int_{\underline{y}+\alpha c}^{y^*_{N-1}} \int_{\underline{y}}^{s-\alpha c} \left[(1-F(s))^{N-1} - (1-F(y^*_{N-1}))^{N-1}\right] f(y_0) dy_0 ds \\ &= \int_{\underline{y}+\alpha c}^{y^*_{N-1}} \left[(1-F(s))^{N-1} - (1-F(y^*_{N-1}))^{N-1}\right] F(s-\alpha c) ds \end{aligned}$$

and the latter is

$$\begin{aligned} U_{early} &= E_{y\sim F}\left[\int_y^{y^*_N} (1-F(s))^{N-1} ds\right] \\ &= \int_{\underline{y}}^{y^*_N} \int_y^{y^*_N} (1-F(s))^{N-1} ds f(y) dy \\ &= \int_{\underline{y}}^{y^*_N} \int_{\underline{y}}^{s} f(y) dy (1-F(s))^{N-1} ds \\ &= \int_{\underline{y}}^{y^*_N} (1-F(s))^{N-1} F(s) ds \end{aligned}$$

Now as $c \to 0$, y^*_{N-1} and y^*_N both go to the smaller of r and $\bar{y}$, which gives us two cases to consider.

If $r < \bar{y}$, then as $c \to 0$,

$$\begin{aligned} U_{late} - U_{early} &= \int_{\underline{y}+\alpha c}^{y^*_{N-1}} \left[(1-F(s))^{N-1} - (1-F(y^*_{N-1}))^{N-1}\right] F(s-\alpha c) ds - \int_{\underline{y}}^{y^*_N} (1-F(s))^{N-1} F(s) ds \\ &\to \int_{\underline{y}}^{r} \left[(1-F(s))^{N-1} - (1-F(r))^{N-1}\right] F(s) ds - \int_{\underline{y}}^{r} (1-F(s))^{N-1} F(s) ds \\ &= -\int_{\underline{y}}^{r} (1-F(r))^{N-1} F(s) ds \qquad < \qquad 0 \end{aligned}$$

so $U_{late} < U_{early}$ for c sufficiently small. If $\overline{y} \leq r$, however, then

$$\begin{aligned} U_{late} - U_{early} &= \int_{\underline{y}+\alpha c}^{y^*_{N-1}} \left[(1-F(s))^{N-1} - (1-F(y^*_{N-1}))^{N-1}\right] F(s-\alpha c) ds - \int_{\underline{y}}^{y^*_N} (1-F(s))^{N-1} F(s) ds \\ &\to \int_{\underline{y}}^{\bar{y}} \left[(1-F(s))^{N-1} - (1-F(\bar{y}))^{N-1}\right] F(s) ds - \int_{\underline{y}}^{\bar{y}} (1-F(s))^{N-1} F(s) ds \qquad = \qquad 0 \end{aligned}$$

To find the sign of $U_{late} - U_{early}$ for c small, we therefore calculate $\frac{d}{dc}\left.(U_{late} - U_{early})\right|_{c=0}$. Differentiating,

$$\begin{aligned}
\tfrac{d}{dc}(U_{late} - U_{early}) \quad &= \quad -\alpha\left[(1-F(\underline{y}+\alpha c))^{N-1} - (1-F(y^*_{N-1}))^{N-1}\right]F(\underline{y}) \\
&\quad + \left[(1-F(y^*_{N-1}))^{N-1} - (1-F(y^*_{N-1}))^{N-1}\right]F(y^*_{N-1}-\alpha c)\tfrac{\partial y^*_{N-1}}{\partial c} \\
&\quad + \textstyle\int_{\underline{y}+\alpha c}^{y^*_{N-1}}(N-1)(1-F(y^*_{N-1}))^{N-2}f(y^*_{N-1})\tfrac{\partial y^*_{N-1}}{\partial c}F(s-\alpha c)ds \\
&\quad -\alpha\textstyle\int_{\underline{y}+\alpha c}^{y^*_{N-1}}\left[(1-F(s))^{N-1} - (1-F(y^*_{N-1}))^{N-1}\right]f(s-\alpha c)ds \\
&\quad -(1-F(y^*_N))^{N-1}F(y^*_N)\tfrac{\partial y^*_N}{\partial c}
\end{aligned}$$

The first two lines are both equal to zero. As noted in the proof of Theorem 2.b, $(1-F(y^*_{N-1}))^{N-2}\frac{\partial y^*_{N-1}}{\partial c}$ and $(1-F(y^*_N))^{N-1}\frac{\partial y^*_N}{\partial c}$ both go to zero as $c \to 0$, leaving

$$\begin{aligned}
\tfrac{d}{dc}\left.(U_{late} - U_{early})\right|_{c=0} \quad &= \quad -\alpha \lim_{c\to 0}\textstyle\int_{\underline{y}+\alpha c}^{y^*_{N-1}}\left[(1-F(s))^{N-1} - (1-F(y^*_{N-1}))^{N-1}\right]f(s-\alpha c)ds \\
&= \quad -\alpha\textstyle\int_{\underline{y}}^{\overline{y}}\left[(1-F(s))^{N-1}\right]f(s)ds \qquad < \qquad 0
\end{aligned}$$

so once again $U_{late} < U_{early}$ for c sufficiently small.

A.9 Proof of Proposition 9 (multiple equilibria)

Symmetric equilibrium with general $\hat{y} \in [y_*, y^*]$

Pick $\hat{y} \in [y_*, y^*]$, and define a function $b_{\hat{y}} : [\underline{y}, \hat{y}] \to \mathbb{R}_+$ by

$$b_{\hat{y}}(y) \quad = \quad y + \frac{c + \int_y^{\hat{y}}(1-F(s))^{N-1}(1-F_0(s-\alpha c))ds}{(1-F(y))^{N-1}(1-F_0(y-\alpha c))}$$

We claim that subcontractors bidding $b_{\hat{y}}(y)$ for $y \leq \hat{y}$, and staying out for $y > \hat{y}$, is an equilibrium, accompanied by appropriate off-equilibrium-path beliefs about the cost of a subcontractor submitting an off-equilibrium-path bid $b > \hat{b} \equiv b_{\hat{y}}(\hat{y})$.

Note that $\hat{b}$, the equilibrium bid of a subcontractor with cost $\hat{y}$, satisfies

$$(\hat{b} - \hat{y})(1-F(\hat{y}))^{N-1}(1-F_0(\hat{y}-\alpha c)) \quad = \quad c$$

so a subcontractor with cost $\hat{y}$ is indifferent between bidding $\hat{b}$ and staying out. A subcontractor with costs $y > \hat{y}$ would lose money by bidding $\hat{b}$, and a subcontractor with costs $y < \hat{y}$ would make money.

A prime contractor will always want to bid after receiving a sub-bid of $\hat{b}$ or less. To see this, note that since $\hat{y} < y^*$, and since $\phi(y) = (r-y)(1-F(y))^{N-1}$ is decreasing in y, we have $\phi(\hat{y}) > c$. It follows that

$$\begin{aligned}
C(\hat{b}, \hat{y}) \quad &= \quad (1-F_0(\hat{y}-\alpha c))\hat{b} + F_0(\hat{y}-\alpha c)\hat{y} \\
&= \quad \hat{y} + \frac{c}{(1-F(\hat{y}))^{N-1}} \quad < \quad \hat{y} + (r - \hat{y}) \quad = \quad r
\end{aligned}$$

so any sub-bid of $\hat{b}$ or less makes it profitable for the general contractor to bid.

Next, we rule out deviations to bids $b > \hat{b}$. By inspection, $\hat{b}$ is increasing in $\hat{y}$, and y_* is defined such that $b_{y_*}(y_*) = r$. Thus, $\hat{b} \geq r$; so if the prime contractor does not expect to reduce her costs (at least in expectation) via bid shopping, a sub-bid above $\hat{b}$ will not induce her to bid (and such a sub-bid will therefore not be a profitable deviation). If the support of F extends upwards at least to r, then we assign off-equilibrium-path beliefs that any bid above $\hat{b}$ must be from a subcontractor with costs at least r. Since expected ex post costs given a bid b from a subcontractor with actual costs y are $(1 - F_0(y - \alpha c))b + F_0(y - \alpha c)y$, this is greater than r and she will not bid. Let $\underline{y}_0$ denote the bottom of the support of F_0. By assumption, if the support of F does not extend upwards to r, then it extends downwards at least to $\underline{y}_0 + \alpha c$. In that case, we assign off-equilibrium-path beliefs that any bid above $\hat{b}$ must be from a subcontractor with costs weakly less than $\underline{y}_0 + \alpha c$. In that case, no post-auction subcontractor will be willing to enter, and the prime contractor will not be able to reduce her cost below the pre-auction bid; once again, her costs are above r, and she will not bid. So in either case, bids above $\hat{b}$ are successfully deterred, since they will not induce the prime contractor to bid.

As in previous proofs, bids below $b_{\hat{y}}(\underline{y})$ are dominated by $b_{\hat{y}}(\underline{y})$, and $b_{\hat{y}}$ is continuous, so there are no holes in its range, so we only need to consider deviations to a different type's equilibrium bid; the proof that such deviations are unprofitable mirrors the proof of Proposition 5.

Ranking the symmetric equilibria by total surplus

Following the same logic as in the proof of Theorem 1, with simple bid shopping, the symmetric equilibrium with entry threshold $\hat{y}$ has expected total surplus

$$W_{\hat{y}} \quad = \quad \int_{\underline{y}}^{\hat{y}} \left[v - y + \int_0^{y - \alpha c} F_0(s) ds \right] N(1 - F(y))^{N-1} f(y) dy - NcF(\hat{y})$$

Differentiating, and then substituting $(r - y^*)(1 - F(y^*))^{N-1}$ for c, gives

$$\begin{aligned} \frac{dW_{shop}}{d\hat{y}} &= \left[v - \hat{y} + \textstyle\int_0^{\hat{y} - \alpha c} F_0(s) ds \right] N(1 - F(\hat{y}))^{N-1} f(\hat{y}) - Ncf(\hat{y}) \\ &= Nf(\hat{y}) \left((1 - F(\hat{y}))^{N-1} \left[v - \hat{y} + \textstyle\int_0^{\hat{y} - \alpha c} F_0(s) ds \right] - (r - y^*)(1 - F(y^*))^{N-1} \right) \end{aligned}$$

Noting that $y^* \geq \hat{y}$ and $(1 - F(y^*))^{N-1} \leq (1 - F(\hat{y}))^{N-1}$, then,

$$\begin{aligned} \frac{dW_{shop}}{d\hat{y}} &\geq Nf(\hat{y}) \left((1 - F(\hat{y}))^{N-1} \left[v - \hat{y} + \textstyle\int_0^{\hat{y} - \alpha c} F_0(s) ds \right] - (r - \hat{y})(1 - F(\hat{y}))^{N-1} \right) \\ &= N(1 - F(\hat{y}))^{N-1} f(\hat{y}) \left[v - r + \textstyle\int_0^{\hat{y} - \alpha c} F_0(s) ds \right] \end{aligned}$$

For $r \leq v$, this is positive, so total surplus increases in the entry threshold, giving the ranking of symmetric equilibria.

These are all the symmetric equilibria

It's worth noting that these are all the symmetric equilibria (modulo the actions of the threshold types). The logic is similar to the proof of uniqueness; any symmetric equilibrium must be weakly monotonic, then strictly monotonic because mass points in bid distributions can't exist; once the entry threshold is chosen, bidders at the threshold must earn zero profits (or else bidders above the threshold would enter), and the bid function is pinned down by the envelope theorem (incentive compatibility). An entry threshold lower than y_* would mean $b_{\hat{y}}(\hat{y}) < r$, which would allow for profitable deviations since an off-equilibrium-path bid below r would still induce a prime contractor bid (since no beliefs could imply ex post costs above the pre-auction sub-bid); and an entry threshold higher than y^* would induce costs above r at the threshold.

A.10 Calculation of total surplus for Table 3

Suppose F and F_0 are both the uniform distribution on $[0,1]$, $\alpha = 0$, and $r = 1$. (This happens to be a knife-edge case where the entry threshold y^* has a simple closed-form expression.) Without bid shopping, expected total surplus is

$$\begin{aligned} W_{noshop} &= \textstyle\int_0^{y^*} (v-s)N(1-F(s))^{N-1} f(s)ds - NF(y^*)c \\ &= \textstyle\int_0^{y^*} N(1-s)^N ds - Ny^*c \\ &= \tfrac{N}{N+1}\left(1-(1-y^*)^{N+1}\right) - Ny^*c \end{aligned}$$

and

$$c \quad = \quad (r-y^*)(1-F(y^*))^{N-1} \quad = \quad (1-y^*)^N \qquad \longrightarrow \qquad y^* \quad = \quad 1 - c^{\frac{1}{N}}$$

giving

$$W_{noshop} \quad = \quad \frac{N}{N+1}\left(1-c^{\frac{N+1}{N}}\right) - Nc\left(1-c^{\frac{1}{N}}\right)$$

With simple bid shopping with entry threshold $\hat{y}$, as calculated above, total surplus is

$$\begin{aligned} W_{shop} &= \textstyle\int_0^{\hat{y}} \left[v - y + \int_0^{y-\alpha c} F_0(s)ds\right] N(1-F(y)^{N-1} f(y)dy - NcF(\hat{y}) \\ &= \textstyle\int_0^{\hat{y}} \left[1-y+\int_0^y s ds\right] N(1-y)^{N-1}dy - Nc\hat{y} \\ &= \textstyle\int_0^{\hat{y}} \left[1-y+\frac{1}{2}y^2\right] N(1-y)^{N-1}dy - Nc\hat{y} \end{aligned}$$

This is easiest to calculate by changing variables to $t = 1-y$, giving

$$\begin{aligned} W_{shop} &= \textstyle\int_{1-\hat{y}}^1 \left[t + \frac{1}{2}(1-t)^2\right] Nt^{N-1}dt - Nc\hat{y} \\ &= \textstyle\int_{1-\hat{y}}^1 \left[\frac{1}{2} + \frac{1}{2}t^2\right] Nt^{N-1}dt - Nc\hat{y} \\ &= \tfrac{N}{2}\left[\tfrac{t^N}{N} + \tfrac{t^{N+2}}{N+2}\right]\Big|_{t=1-\hat{y}}^1 - Nc\hat{y} \\ &= \tfrac{1}{2}\left(1-(1-\hat{y})^N\right) + \tfrac{N}{2(N+2)}\left(1-(1-\hat{y})^{N+2}\right) - Nc\hat{y} \end{aligned}$$

into which we can plug $\hat{y} = y^* = 1 - c^{\frac{1}{N}}$ or $\hat{y} = y_*$, which we can derive as

$$c \quad = \quad (r-y_*)(1-F(y_*))^{N-1}(1-F_0(y_*-\alpha c)) \quad = \quad (1-y_*)^{N+1} \qquad \longrightarrow \qquad y_* \quad = \quad 1-c^{\frac{1}{N+1}}$$

Plugging in parameters gives Table 3 in the text, which suggests that the "worst" bid-shopping equilibrium tends to still increase total surplus (relative to no bid shopping) when c is low, but not when c is high.

Plugging in y_* and y^* and simplifying, we find

$$W_{shop}^{y_*} - W_{noshop} \quad = \quad \frac{1}{(N+2)(N+1)} - \frac{1}{2}c^{\frac{N}{N+1}} + \frac{N(2N+3)}{2(N+2)}c^{\frac{N+2}{N+1}} - \frac{N^2}{N+1}c^{\frac{N+1}{N}}$$

We can verify numerically that for N up to 10,000, this is positive when $c \leq 0.16$, and negative for N up to 10,000 when $c \geq 0.18$.

A.11 Proof of Proposition 10 (NWBR)

Fix a symmetric equilibrium as described in Proposition 9, and let $\hat{y} \in [y_*, y^*)$ be the entry threshold; we will show the equilibrium fails NWBR. Define

$$b_{\max} \quad = \quad \frac{r - F_0(\hat{y} - \alpha c)\hat{y}}{1 - F_0(\hat{y} - \alpha c)},$$

and consider any off-equilibrium-path bid $b \in (b(\hat{y}), b_{\max})$. Let $\mu(y')$ denote the distribution over bidder types describing the common beliefs of a general contractor and her outside subcontractor should the general submit a bid in the prime auction after receiving the pre-auction bid b. We will show that the only belief μ satisfying the NWBR criterion puts all weight on the deviator having cost $\hat{y}$, and that with this belief the deviation becomes strictly profitable for subcontractors with costs near $\hat{y}$.

Our proof has three steps. First, for the NWBR criterion to have any bite, we must establish that there is some PBE supporting the equilibrium path described in the proof of Proposition 9 in which bidding b is a weak best response for some subcontractor type. We do this by showing that there exists a belief μ that puts weight only on types $\underline{y}$ and $\hat{y}$ such that bidding b is weakly profitable to type $\hat{y}$ and strictly unprofitable for all other types. The second step of the proof establishes that in any PBE supporting the equilibrium path, it is never a weak best response for any bidder type $y \neq \hat{y}$ to submit the bid b. The final step shows that when beliefs following the bid b are concentrated on $\hat{y}$ then the deviation is strictly profitable to a sub with cost $\hat{y}$.

As a preliminary, let us describe the optimal strategies for the post auction subcontractor and the general contractor following receipt of the deviating bid b when μ are the beliefs about the type of subcontractor who made it. A post-auction subcontractor with cost y_0 will enter and bid if

$$\int_{y' \geq y_0} (y' - y_0) d\mu(y') \quad \geq \quad \alpha c$$

Let $\underline{y}_0$ be the lowest point in the support of F_0, and define y_m as

$$y_m \quad = \quad \max\left\{ y_0 : \int_{y' \geq y_0} (y' - y_0) d\mu(y') \quad \geq \quad \alpha c \right\}$$

if the set on the right is nonempty, and $y_m = \underline{y}_0$ otherwise. Then entry will occur whenever the post-auction subcontractor has cost below y_m. The prime contractor will then have expected cost

$$C \quad = \quad (1 - F_0(y_m))b + \int_{y_0 \leq y_m} \int_{y'} \max\{y', y_0\} d\mu(y') dF_0(y_0)$$

and will enter and bid C as long as $C < r$, and may randomize in entry if $C = r$.

Step 1: *There exist beliefs μ that make b a weak best response for a pre-auction sub of cost $\widehat{y}$ and a strictly unprofitable deviation for subs of cost $y \neq \widehat{y}$.*

We specify beliefs that put weight $1-\omega$ on $y = \underline{y}$ and weight ω on $y = \hat{y}$, where ω is chosen such that the prime contractor's expected cost equals r. The prime contractor then mixes between entry and non-entry so as to set the expected payoff from this deviation to zero for a subcontractor of type $\hat{y}$. We then show that it is strictly unprofitable for any other type of subcontractor to deviate to the bid b.

Observe that if $\omega = 1$ then the prime contractor's expected cost is strictly below r. Indeed, with these beliefs the entry threshold for the post-auction contractor is $y_m = \hat{y} - \alpha c$, so we have $C = (1 - F_0(\hat{y} - \alpha c))b + F_0(\hat{y} - \alpha c)\hat{y} < r$, as $b < b_{\max}$ and the left-hand side would be equal to r at $b = b_{\max}$. Equally, if $\omega = 0$ the prime contractor's expected cost is strictly above r. Indeed, with those beliefs the post auction subcontractor can recover his entry cost αc only if his cost is $y_0 = \underline{y} - \alpha c$, which by assumption happens with probability zero, so the prime contractor's expected cost is $b > r$. As ω changes, y_m changes continuously, and since F_0 is continuous, so does C; so a value of ω exists giving the prime contractor expected cost r.

Now consider a pre-auction subcontractor with cost $\hat{y}$. With the specified beliefs we have $y_m < \hat{y} - \alpha c$, since otherwise the marginal post-auction entrant would never cover his entry cost. A pre-auction sub with cost $\hat{y}$ thus knows that when he gets bid-shopped he'll lose. His expected payoff is therefore

$$\Pr(bid)(1 - F(\hat{y}))^{N-1}(1 - F_0(y_m))(b - \hat{y}) - c \tag{15}$$

where $\Pr(bid)$ is the probability with which his prime contractor chooses to enter and bid r following receipt of the bid of b. When $\Pr(bid) = 1$ expression (15) is strictly positive, since it's at least $(1 - F(\hat{y}))^{N-1}(1 - F_0(\hat{y} - \alpha c))(b - \hat{y}) - c$ which is strictly positive since $b > b(\hat{y})$. Of course when $\Pr(bid) = 0$ expression (15) is strictly negative. By setting

$$\Pr(bid) \quad = \quad \frac{c}{(1 - F(\hat{y}))^{N-1}(1 - F_0(y_m))(b - \hat{y})} \tag{16}$$

we ensure the deviating bid b gives a pre-auction sub with cost $\hat{y}$ a payoff of zero, making it a weak best response for him.

Next, we show that bidding b *strictly lowers the payoff* of a pre-action subcontractor with cost $y \neq \hat{y}$. When $y > \hat{y}$ this is obvious, for y wins less often than $\hat{y}$ and receives a lower margin whenever he does so. Bidding b thus results in a strictly negative payoff for him.

Consider therefore a sub with cost $y < \hat{y}$. His expected payoff from deviating to bid b is

$$U_D(y) \quad = \quad \Pr(bid)(1 - F(\hat{y}))^{N-1}\left[(1 - F_0(y_m))(b - y) + \int_{\min\{y, y_m\}}^{y_m}(y_0 - y)dF_0(y_0)\right] - c \tag{17}$$

so his gain from deviation is given by

$$\Delta(y) \quad = \quad U_D(y) - \int_y^{\hat{y}}(1 - F(s))^{N-1}(1 - F_0(s - \alpha c))ds \tag{18}$$

Differentiating, when $y \geq y_m$ we get

$$\begin{aligned}\Delta'(y) &= -\Pr(bid)(1 - F(\hat{y}))^{N-1}(1 - F_0(y_m)) + (1 - F(y))^{N-1}(1 - F_0(y - \alpha c)) \\ &= -\frac{c}{b - \widehat{y}} + (1 - F(y))^{N-1}(1 - F_0(y - \alpha c)) \\ &> -\frac{c}{b(\widehat{y}) - \widehat{y}} + (1 - F(\widehat{y}))^{N-1}(1 - F_0(\widehat{y} - \alpha c)) \quad = \quad 0\end{aligned} \tag{19}$$

(where the second equality follows from (16), the inequality because $b > b(\widehat{y})$ and $y < \widehat{y}$, and the final equality because upon multiplication by $b(\widehat{y}) - \widehat{y}$ this is the equilibrium payoff of the marginal pre-auction entrant) and when $y < y_m$ we get

$$\Delta'(y) = -\Pr(bid)(1 - F(\hat{y}))^{N-1}(1 - F_0(y)) + (1 - F(y))^{N-1}(1 - F_0(y - \alpha c)) > 0 \qquad (20)$$

with the inequality because $\Pr(bid) \leq 1$, $(1 - F(\hat{y}))^{N-1} < (1 - F(y))^{N-1}$ (because $y < \hat{y}$), and $1 - F_0(y) \leq 1 - F_0(y - \alpha c)$ (because $y - \alpha c \leq y$). Since $\Delta(\widehat{y}) = 0$ and $\Delta'(y) > 0$ for all $y < \widehat{y}$ we conclude that $\Delta(y) < 0$ for all $y < \widehat{y}$, as was to be demonstrated.

Step 2: *In any PBE supporting the outcome of Proposition 9 for entry threshold $\hat{y}$ it is never a weak best response for any subcontractor of type $y \neq \hat{y}$ to enter and bid b.*

Consider first a subcontractor of cost $y > \hat{y}$ and suppose that entering and bidding b were a weak best response for him. In equilibrium y does not enter, so this deviation earns him a zero payoff. When bidding b a sub with cost $\hat{y}$ wins more often than y, and earns a higher margin whenever he does, so this deviation would earn $\hat{y}$ a strictly positive payoff. As the marginal entrant, a sub of cost $\hat{y}$ has an equilibrium payoff of zero, establishing that bidding b would be a strictly profitable deviation for $\hat{y}$, contradicting equilibrium.

Next, consider a sub with cost $y < \hat{y}$, who in equilibrium enters and earns positive surplus. Suppose that entering and bidding b were a weak best response for y, so that $\Delta(y) = 0$, where $\Delta(y)$ is given by (18). Then we must have $\Delta'(y) \leq 0$, for otherwise a slightly-higher-cost sub would find the bid b to be a strictly profitable deviation. But if $\Delta'(y) \leq 0$ it follows that if $y \geq y_m$ then (from (19))

$$\Pr(bid)(1-F(\hat{y}))^{N-1}(1-F_0(y_m)) \geq (1-F(y))^{N-1}(1-F_0(y-\alpha c)) > (1-F(\hat{y}))^{N-1}(1-F_0(\hat{y}-\alpha c)),$$

and as $b > b(\hat{y})$ and $y < \hat{y}$ this would make b a strictly profitable deviation for a sub with cost $\hat{y}$:

$$\Pr(bid)(1 - F(\hat{y}))^{N-1}(1 - F_0(y_m))(b - \hat{y}) - c > (1 - F(\hat{y}))^{N-1}(1 - F_0(\hat{y} - \alpha c))(b(\hat{y}) - \hat{y}) - c = U(\hat{y})$$

so b would be a strictly profitable deviation for type $\hat{y}$, contradicting equilibrium.

On the other hand, when $y < y_m$, $\Delta'(y) \leq 0$ would require (from (20)) that $\Pr(bid) > 1$, which is impossible; since $\Delta'(y) > 0$, we cannot have $\Delta(y) = 0$ for any $y < y_m$, or we would have $\Delta(y+\varepsilon) > 0$ for some small $\varepsilon > 0$, contradicting equilibrium.

We conclude that entering and bidding b is never a weak best response for any subcontractor of type $y \neq \hat{y}$.

Step 3: *No symmetric equilibrium with $\widehat{y} < y^*$ survives NWBR.*

From the first two steps, the beliefs following receipt of a pre-auction subcontractor bid $b \in (b(\hat{y}), b_{\max})$ must put all weight on $y = \widehat{y}$. In the first step, we showed that these beliefs induce an expected cost of $C < r$ for the prime contractor that receives the bid b. Thus the prime contractor would bid for sure in the prime auction, giving a subcontractor of cost $\hat{y}$ who deviates and bids b an expected payoff of

$$(1-F(\hat{y}))^{N-1}(1-F_0(\hat{y}-\alpha c))(b-\hat{y})-c \quad > \quad (1-F(\hat{y}))^{N-1}(1-F_0(\hat{y}-\alpha c))(b(\hat{y})-\hat{y})-c \quad = \quad 0,$$

contradicting equilibrium.

A.12 Proof of Proposition 11 (simple bid shopping with added costs)

Suppose the incumbent subcontractor incurs a cost of ε when he is replaced after the auction. Let y^{**} solve

$$(r - y^{**} - F_0(y^{**} - \alpha c)\varepsilon)\,(1 - F(y^{**}))^{N-1} \quad = \quad c$$

and for $y < y^{**}$, let

$$b(y) = y + \frac{c + \int_y^{y^{**}} (1 - F(s))^{N-1}(1 - F_0(s - \alpha c))ds}{(1 - F(y))^{N-1}(1 - F_0(y - \alpha c))} + \varepsilon \frac{F_0(y - \alpha c)}{1 - F_0(y - \alpha c)}$$

Note that a bid of $b(y^{**})$ by a subcontractor with cost $y = y^{**}$ induces an expected prime contractor cost of

$$\begin{aligned} C(b(y^{**}), y^{**}) &= (1 - F_0(y^{**} - \alpha c))b(y^{**}) + F_0(y^{**} - \alpha c)y^{**} \\ &= y^{**} + (1 - F_0(y^{**} - \alpha c))(b(y^{**}) - y^{**}) \\ &= y^{**} + \frac{c}{(1 - F(y^{**}))^{N-1}} + \varepsilon F_0(y^{**} - \alpha c) \end{aligned}$$

which is equal to r by the definition of y^{**}. Further, a subcontractor with cost y^{**} who bids $b(y^{**})$ earns expected payoff

$$\begin{aligned} U(b(y^{**}), y^{**}) &= (1 - F(y^{**}))^{N-1} \left[(1 - F_0(y^{**} - \alpha c))(b(y^{**}) - y^{**}) - \varepsilon F_0(y^{**} - \alpha c)\right] - c \\ &= (1 - F(y^{**}))^{N-1} \left[(1 - F_0(y^{**} - \alpha c))(b(y^{**}) - y^{**}) - \varepsilon F_0(y^{**} - \alpha c)\right] - c \\ &= (1 - F(y^{**}))^{N-1} \left[\frac{c}{(1 - F(y^{**}))^{N-1}} + \varepsilon F_0(y^{**} - \alpha c) - \varepsilon F_0(y^{**} - \alpha c)\right] - c \\ &= 0 \end{aligned}$$

Bids above $b(y^{**})$ (with beliefs that $y = y^{**}$) raise the prime contractor's expected costs above r, and hence would induce him not to bid; bids below $b(\underline{y})$ are dominated. The rest of the proof mirrors the proof of Proposition 5.

When the added cost is instead borne by the winning prime contractor, the entry threshold satisfies

$$(r - y^{**} - \varepsilon)(1 - F(y^{**}))^{N-1} = c$$

and the equilibrium bid function is

$$b(y) = y + \frac{c + \int_y^{y^{**}} (1 - F(s))^{N-1}(1 - F_0(s - \alpha c))^{N-1}ds}{(1 - F(y))^{N-1}(1 - F_0(y - \alpha c))}$$

A bid of $b(y^{**})$ by a subcontractor with cost y^{**} gives the prime contractor an expected cost of

$$\begin{aligned} C(b(y^{**}), y^{**}) &= (1 - F_0(y^{**} - \alpha c))b(y^{**}) + F_0(y^{**} - \alpha c)y^{**} + \varepsilon \\ &= y^{**} + (1 - F_0(y^{**} - \alpha c))(b(y^{**}) - y) + \varepsilon \\ &= y^{**} + \frac{c}{(1 - F(y^{**}))^{N-1}} + \varepsilon \end{aligned}$$

which is r by construction; a subcontractor with cost y^{**} who bids $b(y^{**})$ earns expected payoff

$$(1 - F(y^{**}))^{N-1}(1 - F_0(y^{**} - \alpha c))(b(y^{**}) - y^{**}) - c = 0$$

The rest of the proof is the same as above.

A.13 Proof of Proposition 12 (more aggressive bid shopping)

Now let y^{**} solve

$$(r - y^{**} + A(y^{**}))(1 - F(y^{**}))^{N-1} \quad = \quad c$$

and

$$b(y) \quad = \quad y + \frac{c + \int_y^{y^{**}} (1 - F(s))^{N-1}(1 - F_0(s))^K ds}{(1 - F(y))^{N-1}(1 - F_0(y))^K}$$

where

$$A(y) \quad = \quad \int_0^y \left(1 - (1 - F_0(x))^K - K(1 - F_0(x))^{K-1}F_0(x)\right) dx$$

As noted in the text, a prime contractor receiving a bid of $b(y^{**})$ from a subcontractor with costs y^{**} has expected cost

$$\begin{aligned}
C &= (1 - F_0(y^{**}))^K b(y^{**}) + K(1 - F_0(y^{**}))^{K-1}F_0(y^{**})y^{**} \\
&\quad + \int_0^{y^{**}} x\, d\left(1 - (1 - F_0(x))^K - K(1 - F_0(x))^{K-1}F_0(x)\right) \\
&= (1 - F_0(y^{**}))^K \left(y^{**} + \tfrac{c}{(1-F(y^{**}))^{N-1}(1-F_0(y^{**}))^K}\right) + K(1 - F_0(y^{**}))^{K-1}F_0(y^{**})y^{**} \\
&\quad + y^{**}\left(1 - (1 - F_0(y^{**}))^K - K(1 - F_0(y^{**}))^{K-1}F_0(y^{**})\right) \\
&\quad - \int_0^{y^{**}} \left(1 - (1 - F_0(x))^K - K(1 - F_0(x))^{K-1}F_0(x)\right) dx \\
&= y^{**} + \tfrac{c}{(1-F(y^{**}))^{N-1}} - \int_0^{y^{**}} \left(1 - (1 - F_0(x))^K - K(1 - F_0(x))^{K-1}F_0(x)\right) dx
\end{aligned}$$

which is equal to r by construction; a subcontractor with costs y^{**} who bids $b(y^{**})$ earns expected surplus of

$$-c + (1 - F(y^{**}))^{N-1}(1 - F_0(y^{**}))^K(b(y^{**}) - y^{**}) \quad = \quad 0$$

The rest of the proof mirrors the proofs for the costly bid shopping case.

A.14 Proof of Proposition 13 (ex post regret)

As noted in the text, Rule 1 rules out subcontractor bids above r, since they cannot lead to a general contractor bid. The highest-cost subcontractor type to enter must therefore be willing to bid r with the risk of being bid-shopped; this leads to exactly the equilibrium with entry threshold y_*. Under rule 2, the entry cutoff remains y_*, and as noted in the text, subcontractor bids are still uniquely determined by the envelope theorem, and are the same as in the equilibrium with entry threshold y_* and no other constraint. The constraint requiring general contractors not to bid below their cost based on received sub-bids therefore simply inflates prime contractor bids (from their expected cost up to their received sub-bid), giving the winning prime contractor more surplus at the expense of the procurer.

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